Τελική Έκθεση

ΕΡΕΥΝΗΤΙΚΟΥ ΠΡΟΓΡΑΜΜΑΤΟΣ

Οδηγίες για τον Αντισεισμικό Σχεδιασμό Κατασκευών Βιομηχανικού Εξοπλισμού

ΠΑΡΑΡΤΗΜΑΤΑ

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Νοέμβριος 2002

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ΠΕΡΙΕΧΟΜΕΝΑ

ΠΑΡΑΡΤΗΜΑ Α

ΑΝΑΛΥΣΕΙΣ ΓΙΑ ΤΟ ΦΑΙΝΟΜΕΝΟ ΤΟΥ ΚΥΜΑΤΙΣΜΟΥ ΣΕ ΔΕΞΑΜΕΝΕΣ ΕΙΔΙΚΗΣ ΜΟΡΦΗΣ

Στο παράρτημα αυτό παρουσιάζονται τέσσερις εργασίες οι οποίες έχουν δημοσιευθεί ή έχουν σταλεί για δημοσίευση στα πλαίσια του συγκεκριμένου ερευνητικού προγράμματος, και αφορούν την απόκριση ημι-πλήρων δεξαμενών μορφής οριζόντιου κυλίνδρου και σφαίρας σε εξωτερική διέγερση, σε σχέση με το φαινόμενο κυματισμού.

Οι εργασίες είναι

[1] Papaspyrou S., Valougeorgis D. and Karamanos, S. A., "Refined Solutions of Externally Induced Sloshing in Half-Full Spherical Containers", *Journal of Engineering Mechanics*, ASCE, submitted for publication, February 2002.

[2] Papaspyrou S., Valougeorgis D. and Karamanos, S. A., "Damping Effects on Sloshing Response of Horizontal-Cylindrical Liquid Storage Tanks", *4th National Congress on Computational Mechanics*, Patras, Greece, June 2002.

[3] Karamanos, S. A., Papaspyrou S., and Valougeorgis D., "Sloshing Effects in Spherical Vessels and Their Supports", Paper No. 474, *12th European Conference on Earthquake Engineering*, London, UK, September 2002.

[4] Papaspyrou S., Valougeorgis D. and Karamanos, S. A., "Longitudinal sloshing effects in half full horizontal cylindrical vessels", *2nd MIT Conference on Computational Mechanics*, Boston, MA, June 2003.

REFINED SOLUTIONS OF EXTERNALLY INDUCED SLOSHING IN HALF-FULL SPHERICAL CONTAINERS

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ABSTRACT

A mathematical model is developed for calculating liquid sloshing effects in half-full spherical containers under arbitrary external excitation. The velocity potential is expressed in a series form, where each term is the product of a time function and the associated spatial function. Because of the spherical configuration, the problem is not separable and the associated spatial functions are non-orthogonal. Application of the boundary conditions results in a system of ordinary linear differential equations, in the general form of structural dynamics equations of motion. The system is solved numerically, implementing a typical fourth-order Runge-Kutta integration scheme. The proposed simple methodology is capable of predicting sloshing effects in half-full spherical containers under arbitrary external excitation in an accurate manner. Hydrodynamic pressures and horizontal forces on the wall of a spherical container are calculated for real earthquake ground motion data. Viscous effects are included in the present formulation through an appropriate modification of the dynamic free surface condition, and their influence on the response is examined. Finally, it is shown that for the particular case of harmonic excitation, the system of ordinary differential equations results in a system of linear algebraic equations, which yields an elegant semi-analytical solution.

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1. INTRODUCTION

The sloshing problem has been considered as a typical linear eigenvalue problem, which represents the oscillations of the free surface of an ideal liquid inside a container (Lamb, 1945). Those free oscillations are described through a velocity potential function satisfying: (a) the Laplace equation within the fluid, (b) the no-flow condition on the tank wall, and (c) the kinematic and dynamic free-surface conditions. Considering small-amplitude free oscillations (i.e. linearized conditions on the free surface), and assuming a harmonic solution, the above equations result in a typical eigenvalue problem. The solution provides the natural frequencies of fluid oscillation (sloshing frequencies) and the corresponding sloshing modes, which strongly depend on the shape of the container. For rectangular and vertical-cylindrical containers the eigenvalue-sloshing problem can be solved analytically, using separation of variables (Currie, 1974), and the corresponding sloshing modes are mutually orthogonal and uncoupled. For other geometries (e.g. horizontal cylinders or spheres) exact analytical solutions may not be available, and the use of numerical methods becomes necessary.

Quite often, calculation of sloshing frequencies may not be sufficient in engineering applications and the response of the liquid container under external excitation is necessary. In many practical applications in civil, mechanical, aerospace and marine engineering, hydrodynamic pressures and the corresponding forces due to sloshing need to be calculated. In particular, earthquake-induced sloshing has been recognized as an important issue towards safeguarding the structural integrity of liquid storage tanks, and has been the subject of numerous analytical, numerical and experimental works. The pioneering works of Housner (1957, 1963) presented a solution for the hydrodynamic effects in non-deformable vertical cylinders and rectangles. The solution was split in two parts, namely the "impulsive" part and the "convective" part. This concept constitutes the basis for the API 650 standard provisions (Appendix E) for vertical cylindrical tanks (American Petroleum Institute, 2000). Veletsos (1974), Veletsos & Yang (1977), Haroun & Housner (1981), Haroun (1983), have extended this formulation to include the effects of shell deformation, and its interaction with hydrodynamic effects. More recently, the case of uplifting of unanchored tanks as well as soil-structure interaction effects have been studied extensively, in the papers by Peek (1988), Natsiavas (1988), Veletsos & Tang (1990), Malhotra (1995). Notable contributions on the seismic response of anchored and unanchored liquid storage tanks have been presented by Fisher (1979), Rammerstorfer et al. (1988), Fischer et al. (1991), with particular emphasis on design implications. Apparently, those papers constitute the basis for the seismic design provisions concerning vertical cylindrical tanks in Eurocode 8 (EC8 – part 4.3 – Annex A). In addition to the numerous analytical/numerical works, notable experimental contributions on this subject have been reported [Niwa & Clough (1982), Manos & Clough (1982)]. The reader is referred to the review paper of Rammerstorfer et al. (1990) for a thorough presentation and a concise literature review of liquid storage tank response under seismic loads, including fluid-structure and soil-structure interaction effects. In a recent publication, Ibrahim et al. (2001) have reviewed a large number of publications on sloshing dynamics, addressing special issues such as nonlinear sloshing, equivalent mechanical models, stochastic excitation, deformable wall effects, hydrodynamic impact, or sloshing in low gravitation fields.

The above studies have been concentrated almost exclusively on vertical-cylindrical tanks, as well as on rectangular tanks. On the other hand, spherical vessels have significant applications (e.g. in chemical plants and refineries), and their sloshing response under strong seismic events is of particular interest for a reliable estimate of the total horizontal force and the corresponding overturning moment. It is interesting to note that the amount of theoretical and numerical works concerning sloshing in spherical containers is quite limited, when compared with the large number of works in vertical cylinders. Furthermore, in current design practice, the recent provisions of Eurocode 8 (Annex A of EC8, Part 4) are particularly detailed concerning sloshing hydrodynamic effects due to earthquake excitation in rectangles and vertical cylinders, whereas the case of spherical vessel is not considered. Similarly, the API provisions for the seismic design of liquid storage tanks (Appendix E of API standard 650) refer exclusively to vertical cylinders.

Budiansky (1960) has examined sloshing effects in circular canals and spheres, using space transformations to map the initial circular or spherical region to a more convenient plane region. The flow field was described by a set of integral equations, which was solved using a Galerkin-type solution. Numerical values of modal frequencies and hydrodynamic forces were presented for a spherical container. Moiseev & Petrov (1966) described the application of Ritz variational method for the numerical calculation of sloshing frequencies in vessels of various geometries, including the case of a spherical container. Fox & Kuttler (1981, 1983) obtained upper and lower bounds for the values of sloshing frequencies in a semi-circular canal (the two-dimensional analogue of a spherical tank) using conformal mapping and the method of intermediate problems. However, most of the applied conformal mapping techniques require complicated transformations, while the complex variable methods are not applicable in full three-dimensional problems. McIver (1989) considered horizontal cylindrical and spherical containers, filled up to an arbitrary height. Choosing appropriate coordinate systems, so that the container walls and the free surface coincide with coordinate lines or surfaces, McIver reformulated the eigenvalue-sloshing problem in terms of integral equations, which were solved numerically. More recently, McIver & McIver (1993) presented simple analytical methods to obtain upper and lower bounds of sloshing frequencies in horizontal cylinders, which were found to be in good agreement with the results from a boundary element numerical solution.

Generally, the analysis of sloshing in spherical vessels filled up to an arbitrary height requires a numerical solution. However, for the particular case of a half-full sphere it is possible to develop an analytical solution. Evans & Linton (1993) presented a series-type (semi-analytical) solution of the eigenvalue-sloshing problem in hemispherical containers, minimizing the computational effort. Assuming a harmonic solution with respect to time, the velocity potential was expanded in terms of non-orthogonal bounded harmonic spatial functions. Application of the boundary conditions on the tank wall and the free surface resulted in a homogeneous system of algebraic equations, which was solved in terms of the sloshing frequencies.

The present work is aimed at calculating sloshing effects in half-full spherical containers due to external excitation, with particular emphasis on seismic ground motion, extending the analytical formulation proposed by Evans & Linton (1993). In addition, the present formulation takes into account viscous effects through a simplified approach proposed elsewhere (Faltinsen, 1978). The well-known separation-of-variables approach, which works effectively in vertical cylindrical and rectangular configurations, cannot be applied successfully in spherical vessels, since the governing equation with the associated boundary conditions are not separable. The velocity potential is expanded in bounded series in terms of arbitrary time functions and their associated non-orthogonal spatial functions. Moreover, the solution is divided in two parts as suggested by Isaacson & Subbiach (1991): (a) a "uniform motion" part, trivially obtained, representing the liquid motion which follows the external excitation source, and (b) a part related to sloshing, representing the relative fluid motion within the container. Using the above solution methodology, the problem reduces to a system of ordinary linear differential equations, which is solved numerically. Subsequently, it is possible to compute hydrodynamic pressures and the corresponding sloshing forces in half-full spherical containers under arbitrary external excitation in a simple and efficient manner.

The present paper is organized in the following manner. In Section 2, the mathematical formulation and the solution methodology of the half-full sphere response problem under external excitation is described in detail. The important case of harmonic excitation is studied separately in Section 3, and explicit expressions for the hydrodynamic pressures and forces on the vessel are derived. Numerical results are presented in Section 4. The validity and the accuracy of the proposed methodology in terms of the sloshing frequency values are examined first. Subsequently, semi-analytical results for harmonic excitation are derived and, finally, numerical results for half-full spherical containers under arbitrary excitation are presented. In Section 5, a brief summary and some important concluding remarks are stated.

2. THEORETICAL FORMULATION AND SOLUTION

In the present analysis of half-full spherical tanks, the flow is considered incompressible and irrotational. The vessel wall is assumed rigid (non-deformable), since in most applications, spherical vessels are rather thick to resist high levels of internal pressure. The total velocity potential $\Phi=\Phi(x,y,z,t)$ satisfies the Laplace equation within the fluid domain, subjected to the free-surface dynamic and kinematic boundary conditions, and the kinematic condition at the container wall. The container undergoes an arbitrary motion in the direction of a specific axis (say the x axis) with displacement X(t), as shown in Figure 1. The acceleration of the external excitation $\ddot{X}(t)$ and the resulting hydrodynamic force F(t) may be considered as the input and the output of the system, respectively. The amplitude of the external excitation and the resulting free surface elevation (sloshing wave) are assumed to be sufficiently small to allow linearization of the problem. The formulation results in a system of second order ordinary linear differential equations.

2.1 Half-full spherical container under external excitation

The fluid is contained in a rigid spherical vessel of radius R. The vessel is half full, the origin of the coordinate system xyz, is set at the center of the sphere, which is also the center of the free surface disk and the y-axis points vertically downwards. The complete configuration of the system is shown in Figure 1, and the geometry is described in terms of the spherical coordinates

$$x = r\sin\theta\cos\psi, \tag{1a}$$

$$y = r\cos\theta \tag{1b}$$

and

$$z = r\sin\theta\sin\psi \tag{1c}$$

It is assumed that the fluid inside the container is inviscid and the flow can be described by a velocity potential function $\Phi(r,\theta,\psi,t)$, which satisfies Laplace equation

$$\nabla^2\Phi = \frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2\Phi}{\partial\psi^2} = 0 \tag{2}$$

$$r < R,\ 0 \le \theta < \pi/2,\ 0 \le \psi \le 2\pi\,.$$

The velocity potential is also subjected to the linearized dynamic and kinematic free surface conditions

$$\frac{\partial \Phi}{\partial t} - g\eta = 0, \qquad \text{at } \theta = \pi/2,\ r < R,\ 0 \le \psi \le 2\pi \tag{3a}$$

and

$$\frac{1}{r}\frac{\partial \Phi}{\partial \theta} + \frac{\partial \eta}{\partial t} = 0, \qquad \text{at } \theta = \pi/2,\ r < R,\ 0 \le \psi \le 2\pi \tag{3b}$$

respectively, where g is the gravitational constant and $\eta = \eta(r,\psi,t)$ is the free surface elevation. Finally, the sloshing potential should satisfy the kinematic condition at the hemispherical wall of the container

$$\frac{\partial \Phi}{\partial r} = \dot{X}(t)\sin\theta\cos\psi, \qquad \text{at } r = R,\ 0 \le \theta < \pi/2,\ 0 \le \psi \le 2\pi. \tag{4}$$

Subsequently, the velocity potential $\Phi(r,\theta,\psi,t)$ is decomposed in two parts, as suggested by Isaacson & Subbiach (1991)

$$\Phi(r,\theta,\psi,t) = f(r,\theta,\psi,t) + \varphi(r,\theta,\psi,t), \tag{5}$$

where $f(r,\theta,\psi,t)$ and $\varphi(r,\theta,\psi,t)$ are the "uniform motion" velocity potential and the potential related to sloshing respectively. The velocity potential f corresponds to a rigid body motion of the fluid, which follows exactly the motion of the external excitation source, and the velocity potential φ represents the relative motion of the fluid particles within the container due to sloshing.

2.2 Viscous damping

To account for viscous effects in a real fluid, it is assumed that the dissipation occurs only at the free surface. This simplification, suggested elsewhere (Faltinsen, 1978), enables the use of the inviscid potential theory and the introduction of a dissipation mechanism through a slight modification of the dynamic free surface condition. It is assumed that there exists a force, which opposes particle velocity, and this motivates a modification in the dynamic free-surface condition through the addition of a term proportional to the velocity potential. Although this approach is quite artificial, it has been considered as a simple and efficient method to study damped systems with potential theory (Isaacson & Subbiach, 1991). In the aforementioned works, the damping term was considered proportional to the total potential Φ. In the present analysis, this term is assumed proportional to potential φ only. This is justified from the consideration that damping depends on the relative motion of fluid particles (represented by φ), and not by the total fluid motion (represented by Φ). Based on the above discussion, the linearized dynamic free surface condition is written as follows

$$\frac{\partial \Phi}{\partial t} + \nu\, \varphi - g\, \eta = 0\,, \qquad \text{at } \theta = \pi/2\,,\ r < R\,,\ 0 \le \psi \le 2\pi\,, \tag{6}$$

where ν is the viscosity coefficient. Combination of Equations (3b) and (6) leads to the following mixed boundary condition

$$\frac{\partial^2 \Phi}{\partial t^2} + \nu\, \frac{\partial \varphi}{\partial t} + \frac{g}{r}\, \frac{\partial \Phi}{\partial \theta} = 0\,, \qquad \text{at } \theta = \pi/2\,,\ r < R\,,\ 0 \le \psi \le 2\pi\,. \tag{7}$$

2.3 General solution for half-full spherical containers

The externally induced sloshing problem consists of the governing Laplace equation (2), and the boundary conditions (4) and (7). Assuming arbitrary motion of the external source in the x direction (Figure 1) and decomposition of the total potential in the form of Equation (5), the uniform motion potential f is taken as

$$f\left(r,\theta,\psi,t\right) = f(x,t) = \dot{X}(t)\, x = \dot{X}(t)\, r \sin\theta \cos\psi \tag{8}$$

which satisfies the Laplace equation (2), and the following conditions

$$\frac{\partial f}{\partial x}=\dot{X}(t), \qquad \frac{\partial f}{\partial z}=0 \qquad \text{and} \qquad \frac{\partial f}{\partial y}=0. \tag{9}$$

Thus, the unknown potential φ associated with sloshing, should satisfy the Laplace equation (2) within the fluid region and the following boundary conditions:

$$\frac{\partial^2\varphi}{\partial t^2}+\nu\frac{\partial\varphi}{\partial t}+\frac{g}{r}\frac{\partial\varphi}{\partial\theta}=-\ddot{X}(t)\, r\cos\psi, \quad \text{at } \theta=\pi/2,\ r<R,\ 0\le\psi\le 2\pi \tag{10}$$

and

$$\frac{\partial\varphi}{\partial r}=0, \qquad \text{at } r=R,\ 0\le\theta\le\pi/2,\ 0\le\psi\le 2\pi. \tag{11}$$

A solution for the unknown function φ is considered in a series form as

$$\varphi(r,\theta,\psi,t)=\sum_{n=m}^{\infty}\dot{q}_n(t)\,\varphi_n(r,\theta,\psi;m), \qquad r<R,\ 0\le\theta<\pi/2,\ 0\le\psi\le 2\pi. \tag{12}$$

where $q_n(t)$ are unknown arbitrary time functions, and φ_n are the corresponding spatial functions obtained from the general solution of Laplace equation in spherical coordinates, given by

$$\varphi_n(r,\theta,\psi;m)=P_n^m(\cos\theta)\, r^n\cos(m\psi),\ r<R,\ 0\le\theta<\pi/2,\ 0\le\psi\le 2\pi. \tag{13}$$

In the above expression, $P_n^m(\cos\theta)$ (with $m\le n$ and n=0,1,2,3...), are the associated Legendre functions (Kreyszig, 1999). Considering the form of the combined free surface condition, expressed in Equation (10), it can be readily shown that only the terms corresponding to m=1 are nonzero, whereas all other terms vanish. Furthermore, as suggested by Evans & Linton (1993), the expression for the unknown potential is rewritten in the form

$$\varphi(r,\theta,\psi,t)=\sum_{n=1}^{\infty}\left[\dot{q}_{2n-1}(t)\,P_{2n-1}^1(\cos\theta)\,r^{2n-1}+\dot{q}_{2n}(t)\,P_{2n}^1(\cos\theta)\,r^{2n}\right]\cos\psi \tag{14}$$

separating odd and even terms of the series. Substituting Equation (14) into Equation (10) the following relations are obtained:

$$q_2(t)=\frac{1}{3g}\left[\ddot{q}_1(t)+\nu\,\dot{q}_1(t)\right]-\frac{1}{3g}\ddot{X}(t), \tag{15a}$$

and

$$q_{2n}(t)=\frac{1}{(2n+1)g}\left[\ddot{q}_{2n-1}(t)+\nu\,\dot{q}_{2n-1}(t)\right], \qquad \text{for } n>1. \tag{15b}$$

Equations (15) are substituted back into Equation (14) and then the boundary condition at the container wall, expressed by Equation (11), is applied to yield

$$\sum_{n=1}^{\infty}\left\{\frac{2n}{2n+1}\,\frac{R}{g}P_{2n}^{1}(\cos\theta)\,\ddot{q}_{2n-1}(t)+\nu\frac{2n}{2n+1}\,\frac{R}{g}P_{2n}^{1}(\cos\theta)\,\dot{q}_{2n-1}(t)+(2n-1)\,P_{2n-1}^{1}(\cos\theta)\,q_{2n-1}(t)\right\}R^{2n-2}$$

$$=\frac{2}{3}\,\frac{R}{g}P_2^1(\cos\theta)\,\ddot{X}(t)\,. \tag{16}$$

Subsequently, applying the following integral operator on Equation (16)

$$I_s=\int_0^1\cdots\cdots P_{2s-1}^{1}(\mu)\;d\mu, \qquad s=1,2,3,\ldots \tag{17}$$

and conducting some mathematical manipulations, the following infinite system of second-order ordinary linear differential equations is obtained:

$$[\mathbf{M}]\;\{\ddot{q}\}+[\mathbf{C}]\;\{\dot{q}\}+[\mathbf{K}]\;\{q\}=\{\gamma\}\ddot{X}\,. \tag{18}$$

In the above system, $[\mathbf{M}]$ is a non-symmetric square matrix with elements

$$M_{sn}=\frac{2n}{2n+1}\;\frac{R^{2n-1}}{g}\int_0^1 P_{2n}^{1}(\mu)\,P_{2s-1}^{1}(\mu)\;d\mu, \qquad n=1,2,3,\ldots \quad \text{and} \quad s=1,2,3,\ldots \tag{19a}$$

$[\mathbf{C}]$ is a square matrix proportional to $[\mathbf{M}]$

$$[\mathbf{C}] = \nu[\mathbf{M}] \tag{19b}$$

[K] is a diagonal matrix with elements

$$K_{nn} = (2n-1)\, R^{2n-2} \int_0^1 P^1_{2n-1}(\mu)\, P^1_{2n-1}(\mu)\, d\mu \qquad n=1,2,3,\ldots \tag{19c}$$

$\{\gamma\}$ is a vector with elements

$$\gamma_s = \frac{2R}{3\,g} \int_0^1 P^1_2(\mu)\, P^1_{2s-1}(\mu)\, d\mu \qquad s=1,2,3,\ldots \tag{19d}$$

and $\{q\}$ is the unknown vector with components $q_{2n-1}(t)$, n=1,2,... Solution of Equations (18) can be performed through a typical time marching numerical scheme, and leads to the calculation of the arbitrary time functions $q_{2n-1}(t)$ and their first and second derivatives.

The system of Equations (18) is in the regular form of structural dynamics equations of motion. More specifically, $\{q\}$ is the vector of unknown generalized coordinates, [**M**], [**C**] and [**K**] may be considered as the mass, damping and stiffness matrices of the system, respectively, and $\{\gamma\}$ is the vector expressing the contribution (participation) of external excitation on the dynamic equilibrium.

Upon numerical solution of the truncated system of ordinary differential equations in terms of $q_{2n-1}(t)$, functions $q_{2n}(t)$ should be determined, so that the potential φ associated with sloshing is completely defined. To calculate functions $q_{2n}(t)$, it is straightforward to use Equations (15), which express $q_{2n}(t)$ in terms of $\ddot{q}_{2n-1}(t)$, $\dot{q}_{2n-1}(t)$ and the acceleration of the external excitation $\ddot{X}(t)$. Implications may arise when the second derivatives of $q_{2n}(t)$ are calculated, to compute hydrodynamic pressure and forces. This requires calculation of the fourth derivatives of $q_{2n-1}(t)$ and X(t). In the case of seismic input, $\ddot{q}_{2n-1}(t)$ and $\ddot{X}(t)$ are irregular functions, in the form of a ground acceleration seismic record, containing very sharp variations within very small time intervals and their numerical differentiation may lead to erroneous results. It is possible to avoid such a numerical difficulty, under the observation that vector $\{\gamma\}$ consists of the same elements with the first column of matrix [**M**]. Therefore, Equations (18) can be written as follows

$$[\mathbf{M}]\left(\{\ddot{Q}\}+\nu\{\dot{q}\}\right)+[\mathbf{K}]\{q\}=0 \tag{20}$$

where

$$\{\ddot{Q}\}=\begin{bmatrix} \ddot{q}_1-\ddot{X} \\ \ddot{q}_3 \\ \ddot{q}_5 \\ \ddot{q}_7 \\ \vdots \end{bmatrix} \tag{21}$$

On the other hand, conditions (15) can be written

$$\{\overline{q}\}=[\mathbf{D}]\left(\{\ddot{Q}\}+v\{\dot{q}\}\right) \tag{22}$$

where, $\{\overline{q}\}$ is a vector with components $q_{2n}(t)$, and [**D**] is a diagonal matrix with elements

$$D_{kk}=\frac{1}{(2k+1)g}, \qquad k=1,2,3,\ldots \tag{23}$$

Combining Equations (20) and (22), vector $\{\overline{q}\}$ is calculated as follows

$$\{\overline{q}\}=-[\mathbf{D}]\,[\mathbf{M}]^{-1}[\mathbf{K}]\{q\} \tag{24}$$

so that functions $q_{2n}(t)$ are directly expressed in terms of functions $q_{2n-1}(t)$. Thus, the double differentiation of functions $q_{2n}(t)$ becomes a trivial procedure.

2.4 Hydrodynamic pressures and forces

Once the velocity potential φ associated with sloshing is calculated, the hydrodynamic pressure at any location can be computed from the linearized Bernoulli equation

$$P(r,\theta,\psi,t) = -\rho\,\frac{\partial\Phi}{\partial t} = -\rho\,\frac{\partial f}{\partial t} - \rho\,\frac{\partial\varphi}{\partial t}. \tag{25}$$

On the right hand side of Equation (25) the first term is due to the uniform motion potential, while the second term refers to sloshing effects. The total horizontal force acting on the container is obtained by an appropriate integration of the pressure on the hemispherical wall as

$$F = \int_A P(R,\theta,\psi,t)\,(\mathbf{e}_x \cdot \mathbf{n})\,dA, \tag{26}$$

where A is the hemispherical surface, $\mathbf{e}_x$ is the unit vector in the x direction, and $\mathbf{n}$ the outer unit vector normal to A. The total force can be also expressed as a summation of the "uniform motion" force

$$F_U = -\rho \int_A \frac{\partial f}{\partial t}\,(\mathbf{e}_x \cdot \mathbf{n})\,dA \tag{27a}$$

or, using Equation (8),

$$F_U = -\rho\,R^3\,\ddot{X}(t) \int_0^{2\pi}\int_0^{\pi/2} \sin^3\theta\,\cos^2\psi\,d\theta\,d\psi = -\left(\frac{2}{3}\,\pi\,\rho\,R^3\right)\ddot{X}(t) = -M_w\ddot{X} \tag{27b}$$

where M_w is the total liquid mass of the half-full container, and the force associated with sloshing is

$$F_S = -\rho \int_A \frac{\partial\varphi}{\partial t}\,(\mathbf{e}_x \cdot \mathbf{n})\,dA \tag{28a}$$

or, using Equation (14),

$$F_S = -\rho R^3 \pi \sum_{n=1}^{\infty} R^{2n-2}\left\{\ddot{q}_{2n-1}(t)\,Y_{2n-1} + R\,\ddot{q}_{2n}(t)\,Y_{2n}\right\}. \tag{28b}$$

where

$$Y_s = \int_0^{\pi/2} P_s^1(\cos\theta)\ \sin^2\theta\ d\theta, \qquad s=1,2,3,\ldots\ldots \tag{29}$$

Since the pressure is always normal to the wall of the container, the total hydrodynamic force direction always passes through the center of the sphere.

It is interesting to note that the only numerical work required to obtain the solution is related to the solution of a truncated system of ordinary linear differential equations. As shown in Section 4, a relatively small truncation size N ($n \le N$) is adequate to obtain good results.

2.5 A simplified formulation

It is possible to develop a simplified version of the above formulation considering only the first term (N=1) of the series expansion of the potential φ associated with sloshing in Equation (14). In this case, φ is assumed equal to

$$\varphi(r,\theta,\psi,t) = \left[\dot{q}_1\ r\ P_1^1(\cos\theta)\ +\ \dot{q}_2\ r^2\ P_2^1(\cos\theta)\right]\cos\psi\,. \tag{30}$$

Applying the boundary conditions on the free surface and the container wall, the final equation, analogous to Equations (18), is equal to

$$\ddot{q}_1 + \nu\ \dot{q}_1 + \left(\frac{4g}{3R}\right) q_1 = \ddot{X}\,. \tag{31}$$

The other unknown function q_2 is defined by the following equation, analogous to Equation (15a),

$$q_2 = \frac{1}{3g}\left[\ddot{q}_1 + \nu\ \dot{q}_1 - \ddot{X}\right], \tag{32}$$

which results in

$$q_2 = -\left(\frac{4}{9R}\right) q_1, \tag{33}$$

an expression analogous to Equation (24).

It is possible to write Equation (31) in the form of a linear oscillator with viscous damping:

$$\ddot{q}_1 + 2\xi_S \omega_S \dot{q}_1 + \omega_S^2 \, q_1 = \ddot{X} \tag{34}$$

where

$$\omega_S = \sqrt{4g/3R} \tag{35}$$

is the circular (undamped) frequency and

$$\xi_S = \nu / 2\omega_S \tag{36}$$

is the damping ratio. Equation (36) can be employed to estimate the value of viscosity coefficient ν, if the damping ratio ξ_S is somehow estimated (e.g. experimentally). Regarding the ω_S value, it is an approximation of the first sloshing frequency ω_1, since only one term of the series expansion is employed (N=1). The dependence of sloshing frequency values on the truncation size N will be discussed in detail in Section 4.

The hydrodynamic pressure on the wall is calculated from Equation (25) and the corresponding force becomes

$$F = F_U + F_S = -\, M_w \ddot{X} + \frac{M_w}{2} \ddot{q}_1 = -\, M_w \ddot{X} + M_S \, \ddot{q}_1 \tag{37}$$

In the above equation, M_S expresses the part of liquid mass associated with sloshing motion. The form of Equations (34) and (37) motivates the development of a simple mechanical model to describe the sloshing response of a half-full spherical container. Setting

$$\begin{aligned} u_1 &= (-q_1) + X \\ u_2 &= X \end{aligned} \tag{38}$$

it is straightforward to re-write Equations (34) and (37) in the following manner

$$M_S \ddot{u}_1 + \nu M_S \left(\dot{u}_1 - \dot{u}_2\right) + \omega_S^2\, M_S \left(u_1 - u_2\right) = 0 \tag{39}$$

$$F = -M_S\, \ddot{u}_1 - \left(M_w - M_S\right)\ddot{u}_2 = -\frac{M_w}{2}\, \ddot{u}_1 - \frac{M_w}{2}\ddot{u}_2 \tag{40}$$

Based on Equations (39) and (40), the proposed mechanical model is shown in Figure 2. In this model, u_2 represents the motion of the external source, and u_1 expresses the motion of the liquid mass associated with sloshing. In addition, the total liquid mass M_w is split in two equal parts m_1 and m_2, which correspond to u_1 and u_2, and express the so-called "convective" (or "sloshing") mass and "impulsive" motion respectively, a concept introduced by Housner (1957).

3. SOLUTION FOR HARMONIC EXCITATION

The theoretical formulation for arbitrary excitation is significantly simplified when the rigid container undergoes a harmonic motion

$$\dot{X}(t) = U\, e^{-i\omega t}, \tag{41}$$

where U is the velocity amplitude, and ω is the angular frequency of the external excitation source. Assuming steady-state conditions, the total velocity potential $\Phi(x,y,z,t)$ is written as

$$\Phi(x,y,z,t) = [f(x) + \varphi(x,y,z)]\, e^{-i\omega t}, \tag{42}$$

where

$$f(x) = U\, x = U\, r \sin\theta \cos\psi \tag{43}$$

and

$$\varphi(x,y,z) = \sum_{n=1}^{\infty}\left[a_{2n-1}\, P_{2n-1}^{1}(\cos\theta)\, r^{2n-1} + a_{2n}\, P_{2n}^{1}(\cos\theta)\, r^{2n}\right]\cos\psi \tag{44}$$

are the uniform motion potential and the potential related to sloshing respectively. Equation (44) can be readily obtained from Equation (14) assuming harmonic functions for $\dot{q}_n(t)$

$$\dot{q}_n(t)=a_n\ e^{-i\omega t} \tag{45}$$

More specifically, applying the corresponding boundary conditions, the following relations are obtained [compare with Equations (15a) and (15b)]

$$a_2=-\frac{1}{3}\left(\frac{\omega^2+i\nu\omega}{g}\right)a_1+\frac{1}{3}\frac{\omega^2}{g}U \tag{46a}$$

and

$$a_{2n}=-\frac{1}{(2n+1)}\left(\frac{\omega^2+i\nu\omega}{g}\right)a_{2n-1}, \qquad n>1 \tag{46b}$$

Consequently, the following infinite system of linear algebraic equations is obtained

$$\left(-\omega^2[\mathbf{M}]-i\nu\omega[\mathbf{M}]+[\mathbf{K}]\right)\{a\}=-\omega^2\ U\ \{\gamma\}. \tag{47}$$

In the above system, the square matrix [**M**], the diagonal matrix [**K**] and the vector $\{\gamma\}$ are given by Equations (19a), (19c) and (19d) respectively, and $\{a\}$ is the unknown vector with components a_{2n-1}, n=1,2,3... Once the system is solved and the unknown coefficients are computed, the velocity potential φ is determined from Equation (44) using Equations (46). Furthermore, the hydrodynamic pressures and the force acting on the container can be computed from Equations (25) and (26) respectively. It is interesting to note that in the case of harmonic excitation the only computational work required is the solution of a truncated linear algebraic system. Note that when $U=0$ and $\nu=0$, the system of algebraic Equations (47) is reduced to a homogeneous system, which is identical to the one obtained by Evans & Linton (1993).

An estimate of the externally induced sloshing effects on the overall response can be obtained from the computation of the added mass coefficient, defined as follows:

$$C_a = \mathrm{Re}\left[\frac{F_S}{F_U}\right], \tag{48}$$

On the other hand, the dimensionless damping coefficient

$$C_v = \mathrm{Im}\left[\frac{F_S}{F_U}\right] \tag{49}$$

provides a measure of the dissipation mechanisms when viscous effects are included (Isaacson & Subbiach, 1991) In the above expressions, Re[ ] and Im[ ] denote the real and the imaginary part of the F_S/F_U ratio respectively.

It is possible to obtain an elegant analytical solution, if the sloshing potential $\varphi(x,y,z)$ is approximated only with the first term (N=1) of the series expansion. The system of Equations (47) now reduces to a scalar equation in terms of the unknown coefficient a_1. The remaining unknown coefficient a_2 is computed from Equation (46a). The resulting expressions are substituted into Equation (44) to obtain the following closed-form expression for the velocity potential φ

$$\varphi(r,\theta,\psi,t) = \frac{U\omega^2}{\left(\frac{4g}{3R} - \omega^2\right) - i\nu\omega}\left(1 - \frac{4r}{3R}\cos\theta\right) r\sin\theta\cos\psi\, e^{-i\omega t}. \tag{50}$$

Furthermore, the force F_S corresponding to sloshing becomes

$$F_S = i\omega U \frac{1}{3}\pi R^3 \rho \frac{\omega^2}{\left(\frac{4g}{3R} - \omega^2\right) - i\nu\omega} e^{-i\omega t}. \tag{51}$$

Finally, the added mass coefficient and the dimensionless damping coefficient are

$$C_a = \frac{1}{2}\frac{\omega^2\left(\frac{4g}{3R}-\omega^2\right)}{\left(\frac{4g}{3R}-\omega^2\right)^2+(\nu\omega)^2} = \frac{1}{2}\frac{\lambda^2\left(1-\lambda^2\right)}{\left(1-\lambda^2\right)^2+\left(2\lambda\xi_S\right)^2} \tag{52}$$

and

$$C_v = \frac{1}{2}\frac{\nu\omega^3}{\left(\frac{4g}{3R}-\omega^2\right)^2+(\nu\omega)^2} = \frac{1}{2}\frac{2\lambda^3\xi_S}{\left(1-\lambda^2\right)^2+\left(2\lambda\xi_S\right)^2} \tag{50}$$

respectively, where $\lambda=\omega/\omega_S$ is the ratio of the external frequency over the natural frequency of the oscillator.

4. NUMERICAL RESULTS AND DISCUSSION

The numerical results presented in this section are based on the solution of Equations (47) and (18) for the cases of harmonic and arbitrary excitation respectively. The convergence of the solution is always tested, increasing the truncation size of the expansion, so that accurate results up to certain significant figures are obtained. In all cases analyzed, despite the fact that the series solution is expressed in terms of non-orthogonal functions, the convergence of the solution is rapid.

4.1 Sloshing frequencies

To establish some confidence in the present formulation the eigenvalue problem is considered first, assuming no external excitation ($X(t)=0$). The convergence rate and the expected accuracy of the eigenvalue solution are demonstrated numerically, increasing the value of truncation size N. In Figure 3 the variation of the first three eigenvalues ω_1, ω_2 and ω_3, in terms of the truncation size N for zero dissipation $(\nu = 0)$ is shown. The results indicate that the convergence rate is quite rapid. Furthermore, faster convergence is obtained in lower sloshing frequencies. The required truncation size N to obtain accurate results up to three significant figures for ω_1, ω_2 and ω_3 is N=3, N=8 and N=12 respectively. It is interesting to note that the ω_S value is 3.617 [derived analytically Equation (35)], offers a reasonable estimate of the converged value of the first eigenfrequency ω_1 (3.912). It is noted that the converged ω_1, ω_2 and ω_3 values from the present analysis are identical to those reported by Evans & Linton (1993). It is also interesting to note that the value of ω_1 is in very good agreement with experimental results from unpublished tests conducted by Lockheed Missile Systems Division, as reported by Budiansky (1960).

When damping is present ($\nu \neq 0$) the eigenvalues of the system become complex because of energy dissipation effects. The convergence of the complex eigenfrequencies ω_1, ω_2 and ω_3 of the damped system is similar to the corresponding eigenfrequencies of the undamped system. Some typical results for ν=0.72 are shown in Figure 4, where the convergence of the real and imaginary parts of the first and the second sloshing frequency is demonstrated.

It should be underlined that the sloshing frequency values in hemispherical containers depend on the truncation size of the series expansion N, due to the non-orthogonality of the spatial functions $\varphi_n(x,y,z)$ in Equations (12) and (13). On the other hand, when a similar series solution approach [analogous to Equation (12)] is applied in vertical cylinders or rectangles, mutually orthogonal functions $\varphi_n(x,y,z)$ are employed, so that the corresponding sloshing modes are uncoupled (Currie, 1974).

4.2 Hydrodynamic forces under harmonic excitation

The case of harmonic excitation of a half-full sphere is investigated next. The corresponding results are shown in terms of the added mass coefficient C_a and the dimensionless damping coefficient C_v. These coefficients can be used for assessing the effects of sloshing for a wide range of external source frequencies. The convergence of the C_a value is demonstrated for a unit-radius sphere (R=1) and zero damping (v=0) in Tables 1a, 1b and 1c, for different values of the truncation size N and for different values of external excitation frequencies (expressed in terms of ω^2/g). Each Table refers to a range that includes one of the first three natural frequencies of the system, so that the convergence of C_a is examined in the vicinity of the natural frequencies. In the majority of the cases analyzed, the results converge up to three significant figures for $N \leq 10$. As expected, a larger truncation size N is required when the external frequency approaches each of the resonant frequencies of the system.

Subsequently, the added mass coefficient C_a and the dimensionless damping coefficient C_v are plotted as functions of the external excitation frequency (ω^2/g) in Figures 5 and 6 respectively for three different values of the damping parameter ν equal to 0, 0.36 and 0.72 (R=1, g=9.81). According to Equation (36), the three values of v correspond to 0%, 5% and 10% values of damping ratio ξ_S respectively. Figure 5 shows that for the case of zero damping, the response is characterized by large increases in the added mass coefficient C_a in the vicinity of resonant frequencies. There is a sign reversal in C_a at each resonant frequency. When $C_a \leq 0$ the sloshing force F_S opposes the "uniform motion" force F_U resulting in a reduction of the total force amplitude. The extreme values of the added mass coefficient close to the resonant frequencies are significantly reduced when damping is present, and the resonant effect of the higher natural frequencies almost disappears. The large values of C_a for a wide range of excitation frequencies indicate the significant effects of hydrodynamic sloshing on the overall response. Figure 6 presents the corresponding results for the dimensionless damping coefficient C_v. The C_v value exhibits a peak near the first resonant frequency, and much smaller peaks for the higher resonant frequencies. When the damping parameter value is increased, the peaks become smaller and smoother.

4.3 Results for earthquake ground motion

The response of hemispherical liquid containers under earthquake excitation is of particular importance for the seismic analysis of spherical pressure vessels used in refineries and petrochemical industries. In the present paper, the seismic ground motion occurred in Kozani, Greece, in 1995, is considered (Figure 7). The linear system of ordinary differential equations is integrated in time by implementing a fourth-order Runge-Kutta scheme in Matlab programming. Following a short parametric study, the time step Δt is chosen equal to 0.005 sec.

The dependence of the value of the maximum total force F_{max} on the truncation size N is indicated in Table 2. The results indicate that consideration of sloshing in the analysis has a very significant effect on the maximum value of the total force F_{max}. It is also shown that consideration of few terms of the series solution (e.g. N=2) is adequate to provide quite accurate results in terms of the F_{max} value, for engineering purposes.

Figures 8, 9 and 10 show the variation of the uniform force F_U, the force associate with sloshing F_S and the total force F respectively, for a half-full sphere subjected to the Kozani earthquake and for zero damping (v=0). The sphere has a radius R equal to 10 meters, and contains a liquid of density ρ equal to 1000 kgr/m^3 (g=9.81m/sec^2). A truncation size N equal to 2 is considered in this analysis. The results show that the maximum value of the uniform motion force F_{Umax} is significantly larger than the maximum value of the total force F_{max} and that sloshing effects result in a reduction of the total hydrodynamic force. The fact that the dominant earthquake frequencies are significantly larger than the sloshing frequencies offers a reasonable explanation for the beneficial effect of sloshing. More specifically, under these conditions the sloshing force F_S opposes the uniform motion force F_U and, therefore, the total force is reduced.

Figure 11 shows the half-full vessel response in terms of the sloshing force under the Kozani earthquake, for 10% damping (the value of v is chosen equal to 0.228, so that ξ_S=10%). Due to the high values of dominant earthquake frequencies as opposed to the low values of the sloshing frequencies, the sloshing force F_{Smax} is found equal to 2.402 MN. Comparison with the corresponding maximum sloshing force obtained from the analysis of the undamped system (2.408 MN) indicates that the maximum sloshing force is almost unaffected by the presence of damping.

A more detailed presentation of damping effects is demonstrated in Figures 12 and 13, where the time history of the generalized coordinates $q_1(t)$ and $q_3(t)$ for zero damping (v=0) and for 10% damping (v=0.228) is shown. The presence of damping results in a significant attenuation of the $q_1(t)$ and $q_3(t)$ values.

5. Conclusions

A mathematical model is developed for externally induced liquid sloshing in half-full spherical containers. The velocity potential is split in two parts, a "uniform motion" potential (trivially obtained) and a potential associate with sloshing. In this configuration, the problem formulation is not separable and the general solution of the sloshing potential is written as a series expansion of arbitrary time functions and its associated non-orthogonal spatial functions.

Furthermore, viscous damping effects can be taken into consideration through an appropriate modification of the dynamic free surface boundary condition. The formulation reduces in a system of linear differential equations, which is solved numerically.

The present formulation enables the prediction of sloshing effects in hemispherical liquid containers under any form of external excitation, in a simple and efficient manner. The problem is significantly simplified if only the first term of the series is considered, and a mechanical model is proposed to describe sloshing response. For the particular case of harmonic external source, closed-from expressions for the sloshing potential and the sloshing force are obtained.

A numerical investigation of convergence is conducted to determine the sensitivity of results on the truncation size of the series solution. It is found that higher sloshing frequencies require a larger truncation size to achieve convergence. In the case of harmonic excitation, the results are expressed in terms of the added force coefficient and the dimensionless damping coefficient, and indicate that convergence is less rapid in the vicinity of resonant frequencies, and that the presence of damping diminishes resonant effects.

Subsequently, the response of a spherical vessel subjected to a real seismic event is examined, and the total horizontal force acting on the container is calculated. The results indicate that sloshing has a significant effect on the value of the total force. On the other hand, the numerical results demonstrate that few sloshing terms in the series solution are adequate so that very good results are obtained. It is also found that consideration of sloshing results in a reduction of the total force with respect to the "uniform motion" force, because the dominant earthquake frequencies are significantly higher than the sloshing frequencies. For the same reason, the effects of viscous damping have an insignificant effect on the maximum value of the sloshing force.

ACKNOWLEDGMENTS

This work has been partially supported by the Earthquake Planning & Protection Organization (EPPO), Athens, Greece. The authors would also like to thank Dr. V. Lekidis and Dr. Ch. Carakostas, Institute of Engineering Seismology & Earthquake Engineering (ITSAK), Thessaloniki, Greece, for providing the seismic input data of the Kozani earthquake.

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N \ ω^2/g	**1**	**1.3**	**1.6**	**1.9**
1	1.5000	19.500	-3.0000	-1.6764
2	1.0516	2.8712	-53.400	-3.9075
3	1.0476	2.9580	-20.543	-3.1687
4	1.0430	2.9268	-22.199	-3.2180
5	1.0410	2.9160	-22.644	-3.2228
10	1.0393	2.9044	-23.176	-3.2299
15	1.0391	2.9031	-23.241	-3.2307
20	1.0390	2.9027	-23.254	-3.2309

Table 1a: Convergence of C_a versus N (ν=0, R=1). The converged value of ω_1^2/g is 1.5601.

N \ ω^2/g	**4.8**	**5.1**	**5.4**	**5.7**
1	-0.6923	-0.6769	-0.6639	-0.6526
2	-0.7440	-0.7350	-0.7258	-0.7071
3	-0.7588	-0.7467	-0.7378	-0.7304
4	-0.7950	-0.7816	-0.7668	-0.7537
5	-0.7357	-0.6948	-0.6926	-0.7087
10	-0.7049	-0.4008	-1.4348	-0.9841
15	-0.7050	-0.4030	-1.4375	-0.9840
20	-0.7050	-0.4037	-1.4383	-0.9839

Table 1b: Convergence of C_a versus N (ν=0, R=1). The converged value of ω_2^2/g is 5.2753.

N \ ω^2/g	**8.1**	**8.4**	**8.7**	**9.0**
1	-0.5985	-0.5943	-0.5904	-0.5869
2	-0.6665	-0.6621	-0.6581	-0.6544
3	-0.6881	-0.6841	-0.6803	-0.6768
4	-0.6986	-0.6946	-0.6908	-0.6873
5	-0.7049	-0.7010	-0.6972	-0.6937
10	-0.6871	-0.6255	-0.5747	-0.6247
15	-0.6800	-0.4433	-0.9023	-0.7970
20	-0.6801	-0.4450	-0.9023	-0.7969

Table 1c: Convergence of C_a versus N (ν=0, R=1). The converged value of ω_3^2/g is 8.5040.

N	F (max) [MN]
0	4.365
1	2.201
2	1.958
3	1.874
4	1.870

Table 2: Dependence of the maximum total force value (F_{max}) in terms of the truncation size N, (undamped system).

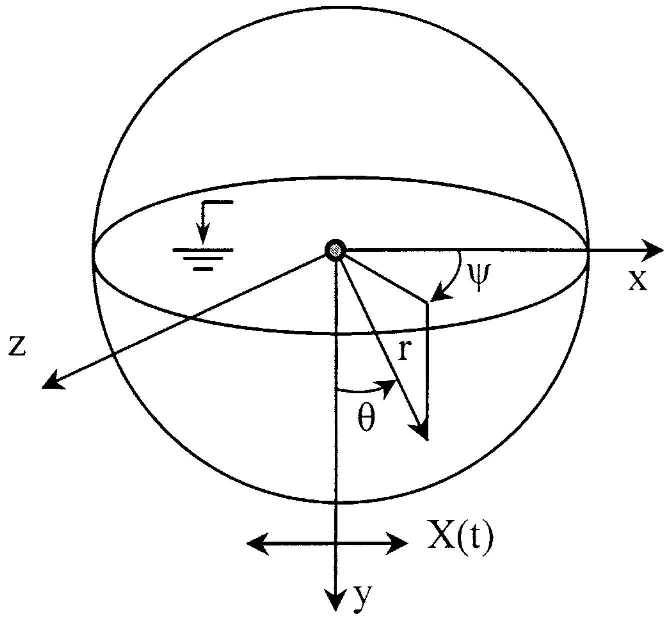


Figure 1: Configuration of half-full spherical container.

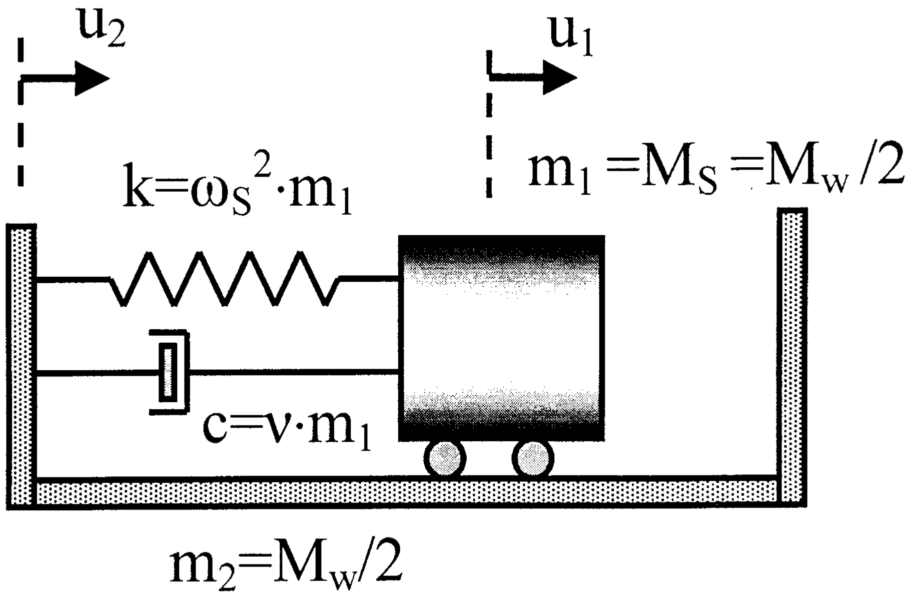


Figure 2: Mechanical model based on the simplified formulation (N=1).

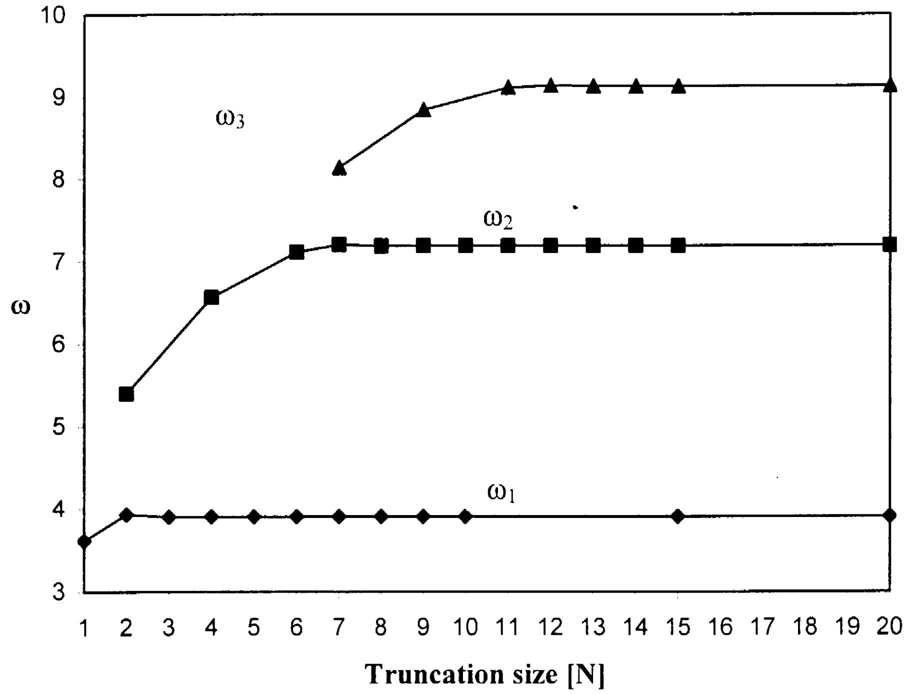


Figure 3: Variation of the first three eigenfrequencies with respect to the truncation size N for v=0, (R=1, g=9.81).

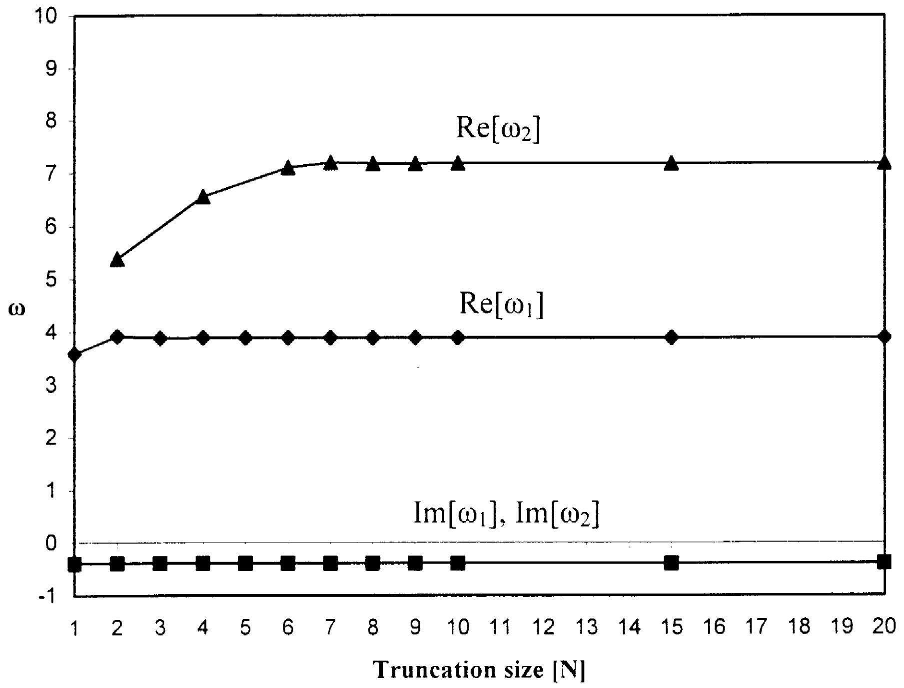


Figure 4: Variation of the first and second eigenfrequency with respect to the truncation size N for v=0.72, (R=1, g=9.81).

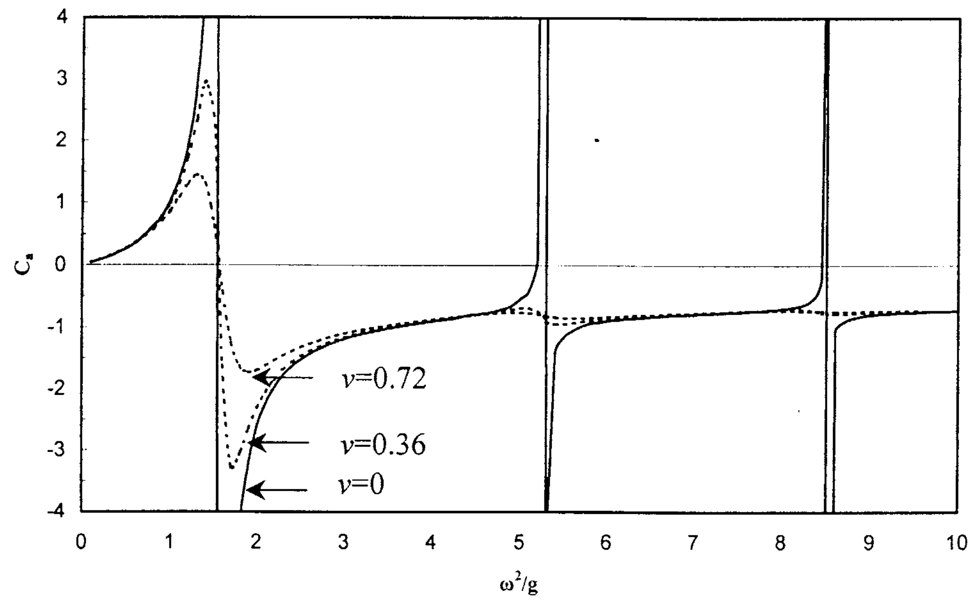


Figure 5: Converged value of C_a in terms of external excitation frequency (ω^2/g) for N=50, (R=1).

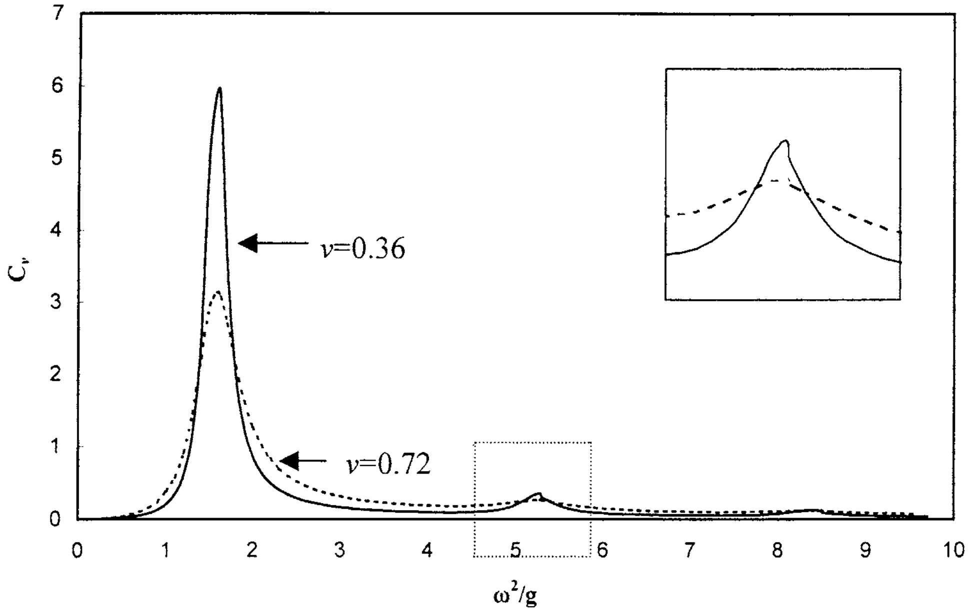
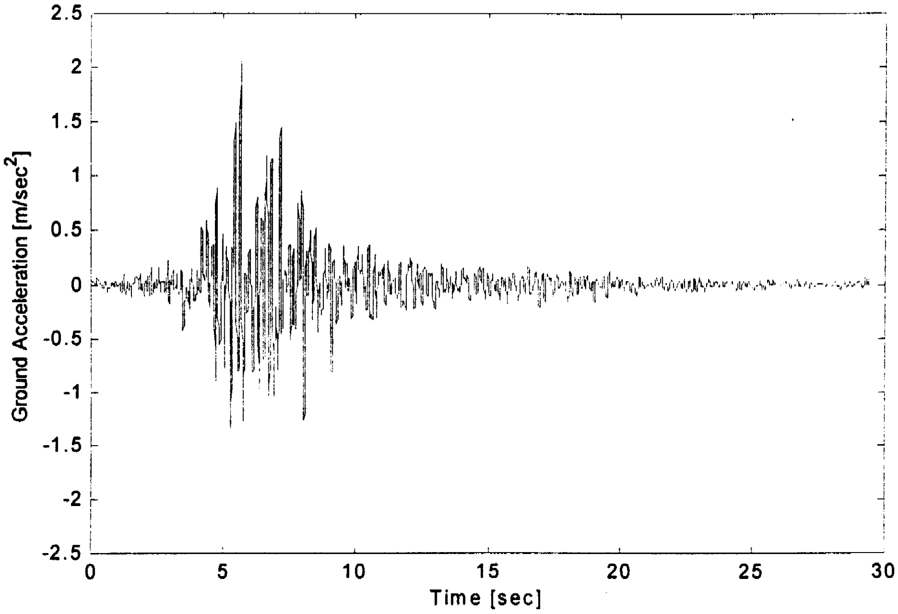


Figure 6: Converged value of C_v in terms of external excitation frequency (ω^2/g) for N=50, (R=1).



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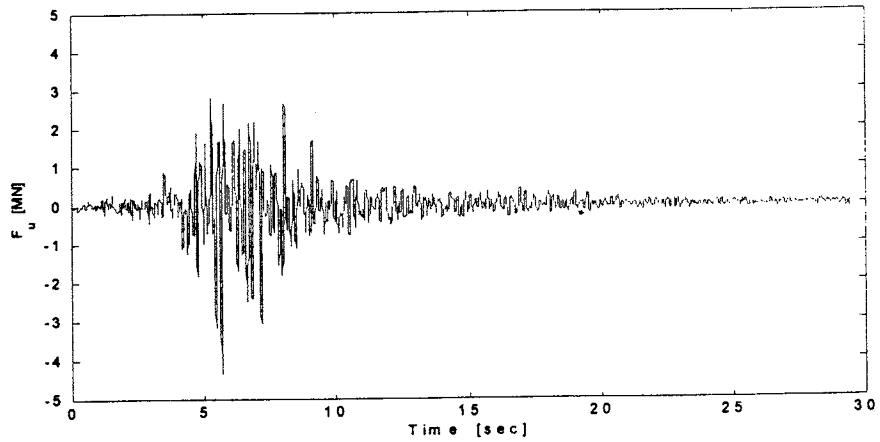

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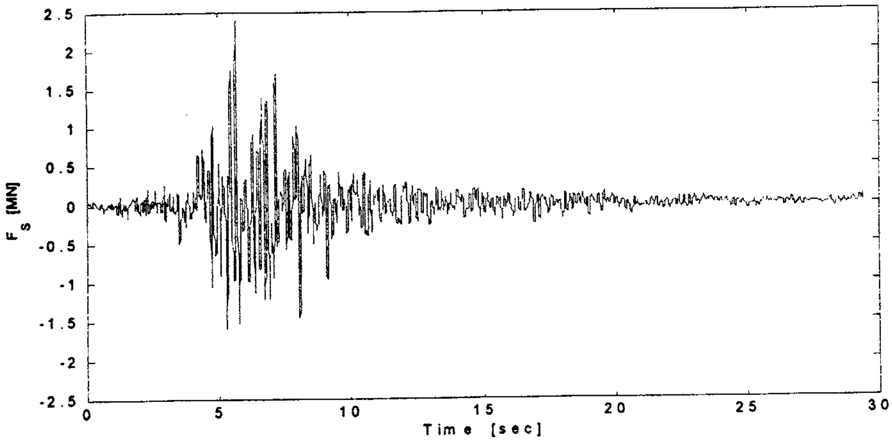


Figure 9: Force associated with sloshing for ν=0 (undamped system) and truncation size N=2, (R=10m, g=9.81m/sec^2, ρ=1000kgr/m^3)

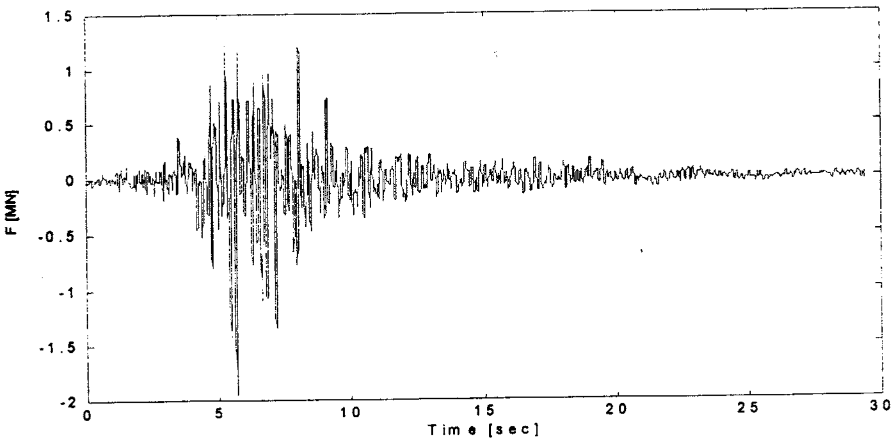
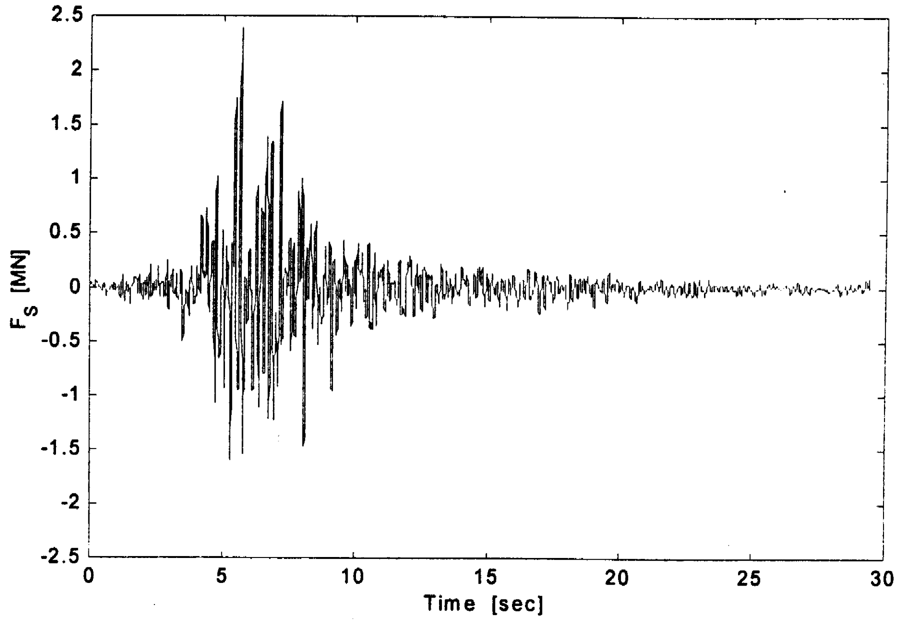


Figure 10: Total force for ν=0 (undamped system) and truncation size N=2, (R=10m, g=9.81m/sec^2, ρ=1000kgr/m^3).



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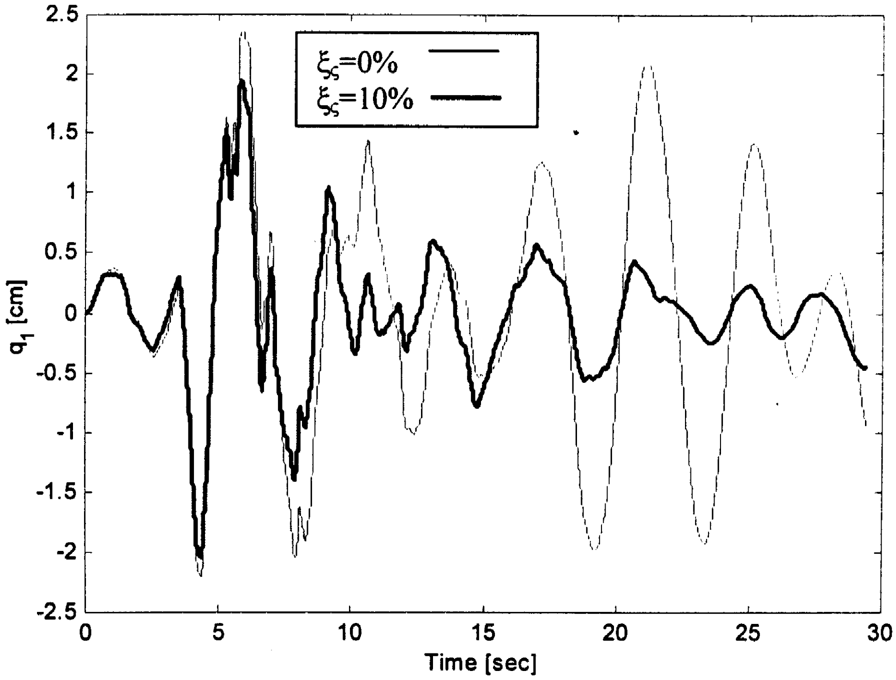

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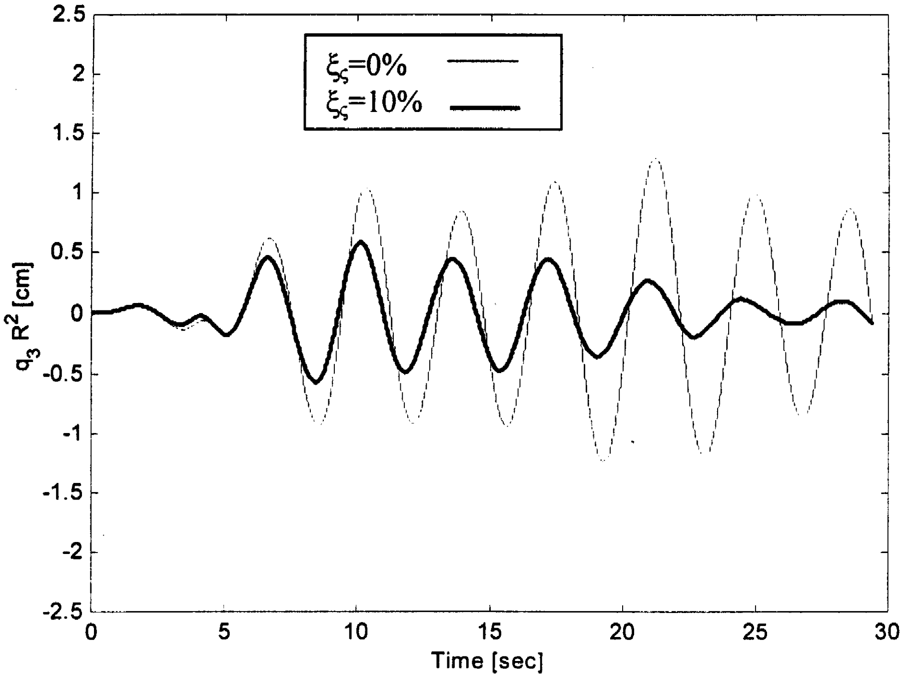

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4th GRACM Congress on Computational Mechanics
GRACM 2002
Patra, 27-29 June, 2002

EXTERNALLY INDUCED SLOSHING IN HORIZONTAL CYLINDRICAL VESSELS WITH ENERGY DISSIPATION

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Keywords: sloshing, horizontal cylindrical vessel, energy dissipation, seismic response.

Abstract. *A mathematical model is developed for sloshing effects in half-full horizontal-cylindrical vessel containing viscous liquid, under arbitrary external excitation. The velocity potential is expressed in a series form, where each term is the product of a time function and the associated spatial function. In this configuration, the problem is not separable and the associated spatial functions are non-orthogonal. Application of the boundary conditions results in a system of ordinary linear differential equations, which are solved numerically. The motion of a real liquid gives rise to energy dissipation, which is modeled through an appropriate modification of the dynamic free-surface condition. Hydrodynamic pressures and horizontal forces on the wall of a horizontal cylindrical container are calculated for harmonic excitation and for a real seismic motion event.*

1. INTRODUCTION

The sloshing problem has been considered as a typical linear eigenvalue problem, which represents the oscillations of the free surface of an ideal liquid inside a container[9]. Those free oscillations are described through a velocity potential function satisfying: (a) the Laplace equation within the fluid, (b) the kinematic condition on the tank wall, and (c) the linearized kinematic and dynamic free-surface conditions for small amplitude oscillations. The solution provides the natural frequencies of fluid oscillation (sloshing frequencies) and the corresponding sloshing modes. The solution strongly depends on the shape of the container. For rectangular and vertical-cylindrical containers the sloshing problem can be solved analytically, using separation of variables[2], and the corresponding sloshing modes are mutually orthogonal and uncoupled. For other geometries (e.g. horizontal cylinders or spheres[13]) exact analytical solutions may not be available, and the use of numerical methods becomes necessary.

Budiansky[1] has examined sloshing effects in circular canals and spheres, numerical values of modal frequencies and hydrodynamic forces were presented for a circular canal and spherical container. Moiseev & Petrov[12] described the application of Ritz variational method for the numerical calculation of sloshing frequencies in vessels of various geometries, including the case of a horizontal cylindrical and spherical container. Fox & Kuttler[5, 6] obtained upper and lower bounds for the values of sloshing frequencies in a semi-circular canal (the two-dimensional analogue of a spherical tank) using conformal mapping and the method of intermediate problems. McIver[10] considered horizontal cylindrical and spherical containers, filled up to an arbitrary height, reformulating the eigenvalue-sloshing problem in terms of integral equations, which were solved numerically. More recently, McIver & McIver[11] presented simple analytical methods to obtain upper and lower bounds of sloshing frequencies in horizontal cylinders.

Generally, the analysis of sloshing in horizontal cylindrical and spherical vessels filled up to an arbitrary height requires a numerical solution. However, for the particular case of a half-full horizontal cylinder and sphere it is possible to develop an analytical solution. Evans & Linton[3] presented a series-type (semi-analytical) solution of the eigenvalue-sloshing problem in hemispherical containers, expanding the velocity potential in terms of non-orthogonal bounded harmonic spatial functions.

The present work is aimed at calculating sloshing effects in half-full horizontal cylindrical containers due to external excitation, extending the analytical formulation proposed by Evans & Linton[3]. Moreover the solution is divided in two parts as suggested by Isaacson & Subbiach[8]: (a) a "uniform motion" part, trivially obtained, representing the liquid motion which follows the external excitation source, and (b) a part related to sloshing, representing the relative fluid motion within the container. In addition, the present formulation takes into account viscous effects through a simplified approach proposed elsewhere[4]. The velocity potential is expanded in bounded series in terms of arbitrary time functions and their associated non-orthogonal spatial functions. The problem reduces to a system of ordinary linear differential equations, which is solved numerically. Subsequently, hydrodynamic pressures and the corresponding sloshing forces are computed in a simple and efficient manner. A similar formulation has been developed recently for spherical vessels[13].

2. THEORETICAL FORMULATION AND SOLUTION

The fluid is contained in a horizontal cylindrical vessel of radius R and the vessel wall is assumed rigid (non-deformable). The vessel is half full with the y-axis of the coordinate system xyz pointing vertically downwards (Figure 1). The geometry is described in terms of the cylindrical coordinates r, θ, z. The container undergoes an arbitrary motion in the direction of a specific axis (say the x axis) with displacement X(t), as shown in Figure 1. The amplitude of the external excitation and the resulting free surface elevation are assumed to be sufficiently small to allow linearization of the problem.

It is assumed that the fluid inside the container is inviscid and the flow can be described by a velocity potential function $\Phi(r,\theta,z,t)$, which satisfies Laplace equation within the fluid volume. In addition, the horizontal cylinder is assumed very long (L>>R) so that the end effects are negligible and all cross-sections normal to the z-axis have the same response, yielding a two-dimensional problem (independent of z). Therefore,

$$\nabla^2\Phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}=0 \qquad r<R,\ -\pi/2<\theta<\pi/2 \tag{1}$$

The velocity potential is also subjected to the linearized dynamic and kinematic free surface conditions

$$\frac{\partial\Phi}{\partial t} - g\eta = 0, \quad \text{and} \quad \pm\frac{1}{r}\frac{\partial\Phi}{\partial\theta}+\frac{\partial\eta}{\partial t}=0 \qquad \text{at } \theta=\pm\pi/2,\ r<R \tag{2}$$

respectively, where g is the gravitational constant and $\eta=\eta(r,\theta,z,t)$ is the free surface elevation. Finally, the sloshing potential should satisfy the kinematic condition at the container wall

$$\frac{\partial\Phi}{\partial r} = \dot{X}(t)\sin\theta, \qquad \text{at } r=R,\ -\pi/2\le\theta\le\pi/2 \tag{3}$$

Subsequently, Φ is decomposed in two parts, as suggested by Isaacson & Subbiach[8]

$$\Phi(r,\theta,t) = f(r,\theta,t)+\varphi(r,\theta,t), \tag{4}$$

where $f(r,\theta,t)$, $\varphi(r,\theta,t)$ are the "uniform motion" velocity potential and the potential related to sloshing respectively. The velocity potential f corresponds to a rigid body motion of the fluid, which follows exactly the motion of the external excitation source, and the velocity potential φ represents the relative motion of the fluid particles within the container due to sloshing.

To account for viscous effects in a real fluid, it is assumed that the dissipation occurs only at the free surface. This simplification, suggested elsewhere[4], enables the introduction of a dissipation mechanism through a slight modification of the dynamic free surface condition

$$\frac{\partial\Phi}{\partial t}+\nu\,\varphi-g\,\eta=0, \qquad \text{at } \theta=\pm\pi/2,\ r<R \tag{5}$$

Combination of Equations (2) and (3) for cylindrical vessel leads to the following mixed boundary condition

$$\frac{\partial^2\Phi}{\partial t^2}+\nu\frac{\partial\varphi}{\partial t}\pm\frac{g}{r}\frac{\partial\Phi}{\partial\theta}=0, \qquad \text{at } \theta=\pm\pi/2,\ r<R \tag{6}$$

Assuming motion of the external source in the (transverse) x direction (Figure 1) and decomposition of the total potential in the form of Equation (4), the uniform motion potential f is taken as

$$f=\dot{X}(t)\,x=\dot{X}(t)\,r\sin\theta \tag{7}$$

which satisfies the Laplace equation (1), and the kinematic conditions at the container wall. Thus, the unknown potential φ associated with sloshing, should satisfy the Laplace equation (1) within the fluid region and the following boundary conditions:

$$\frac{\partial^2\varphi}{\partial t^2}+\nu\frac{\partial\varphi}{\partial t}\pm\frac{g}{r}\frac{\partial\varphi}{\partial\theta}=\mp\ddot{X}\,r, \qquad \theta=\pm\pi/2,\ r<R \tag{8}$$

and

$$\frac{\partial\varphi}{\partial r}=0, \qquad \text{at } r=R,\ -\pi/2\le\theta\le\pi/2,\ 0<z<L \tag{9}$$

A solution for the unknown function φ is considered in a series form as

$$\varphi=\sum_{n=m}^{\infty}\dot{q}_n(t)\,\varphi_n(r,\theta)=\sum_{n=m}^{\infty}\dot{q}_n(t)\,r^n\cos(n\theta), \qquad r<R,\ -\pi/2\le\theta\le\pi/2 \tag{10}$$

where $q_n(t)$ are unknown arbitrary time functions, and φ_n are the corresponding spatial functions. Furthermore, as suggested by Evans & Linton[3] the expression for the unknown potential is rewritten in the form

$$\varphi(r,\theta,t)=\sum_{n=1}^{\infty}\left[\dot{q}_{2n-1}(t)\,r^{2n-1}\sin(2n-1)\theta+\dot{q}_{2n}(t)\,r^{2n}\sin 2n\theta\right] \tag{11}$$

separating odd and even terms of series. Substituting (11) into (8) the following relations are obtained:

$$q_2(t)=\frac{1}{2g}\left[\ddot{q}_1(t)+\nu\dot{q}_1(t)\right]+\frac{1}{2g}\ddot{X}(t), \tag{12a}$$

and

$$q_{2n}(t)=\frac{1}{2ng}\left[\ddot{q}_{2n-1}(t)+\nu\,\dot{q}_{2n-1}(t)\right], \qquad \text{for } n>1. \tag{12b}$$

Equations (12) are substituted back into (11) and then applying the boundary condition at the container wall, expressed by (9), yields

$$\sum_{n=1}^{\infty}\left\{\frac{R^{2n-1}}{g}\sin 2n\theta\,\ddot{q}_{2n-1}(t)+\nu\frac{R^{2n-1}}{g}\sin 2n\theta\,\dot{q}_{2n-1}(t)+(2n-1)\,R^{2n-2}\sin(2n-1)\theta\,q_{2n-1}\right\}$$
$$=-\frac{R}{g}\sin 2n\theta\,\ddot{X}(t) \tag{13}$$

Subsequently, applying the following integral operator on (13)

$$I_s=\int_0^{\pi/2}\ldots\sin(2s-1)\theta\,d\theta, \qquad s=1,2,3\ldots \tag{14}$$

and conducting some mathematical manipulations, the following infinite system of second-order ordinary linear differential equation is obtained:

$$[\mathbf{M}]\ \{\ddot{q}\}+\nu[\mathbf{M}]\ \{\dot{q}\}+[\mathbf{K}]\ \{q\}=-\{\gamma\}\ddot{X} \tag{15}$$

where $[\mathbf{M}]$ is a non-symmetric square matrix, $[\mathbf{K}]$ is a diagonal matrix and $\{\gamma\}$ is a vector with elements

$$M_{sn}=\frac{2n(-1)^{n+s}}{n^2-(s-\frac{1}{2})^2}R^{2n-1} \qquad n=1,2,3\ldots\text{and } s=1,2,3\ldots \tag{16a}$$

$$K_{nn}=(2n-1)\pi g R^{2n-2} \qquad n=1,2,3... \tag{16b}$$

$$\gamma_s=\frac{8(-1)^{s+1}}{3+4s-4s^2}R \qquad S=1,2,3... \tag{16c}$$

and $\{q\}$ is the unknown vector with components $q_{2n-1}(t)$, n=1,2,... Solution of (15) can be performed through a typical time marching numerical scheme, and leads to the calculation of the arbitrary time functions $q_{2n-1}(t)$ and their first and second derivatives. Upon numerical solution of the truncated system of ordinary differential equations in terms of $q_{2n-1}(t)$, functions $\dot{q}_{2n}(t)$ should also be determined from (12).

Once the velocity potential φ associated with sloshing is calculated, the hydrodynamic pressure at any location can be computed from the linearized Bernoulli equation and the total horizontal force acting on the container is obtained by an appropriate integration of the pressure on the hemispherical wall. The total force can be also expressed as a summation of the "uniform motion" force F_U and the force associated with sloshing F_S

$$F_U=-\rho\int_A \frac{\partial f}{\partial t}(\mathbf{e}_x\cdot\mathbf{n})dA \qquad \text{and} \qquad F_S=-\rho\int_A \frac{\partial \varphi}{\partial t}(\mathbf{e}_x\cdot\mathbf{n})dA \tag{17a}$$

Using (7) and (11), F_U and F_S are calculated as follows

$$F_U=-\rho\,\ddot{X}(t)\,R^2\int_0^L\int_{-\pi/2}^{\pi/2}\sin^2\theta\,d\theta\,dz=-\left(\rho\frac{\pi R^2}{2}L\right)\ddot{X}(t)=-M_w\,\ddot{X}(t) \tag{17b}$$

$$F_S=-\rho\,L\,R^2\sum_{n=1}^{\infty}R^{2n-2}\left[\ddot{q}_{2n-1}(t)\,Y_{2n-1}+R\,\ddot{q}_{2n}(t)\,Y_{2n}\right] \tag{17c}$$

where, M_w is the total liquid mass of the half-full horizontal cylinder container and

$$Y_s=\int_{-\pi/2}^{\pi/2}\sin 2s\theta\,\sin\theta\,d\theta, \qquad s=1,2,3... \tag{18}$$

Since the pressure is always normal to the wall of the container, the total hydrodynamic force direction always passes through the center of the cross-section of horizontal cylinder.

It is possible to develop a simplified version of the above formulation considering only the first term (N=1) of the series expansion of the potential φ associated with sloshing in (11) and (17). Then analytical expressions for the hydrodynamic pressure on the wall are calculated and the corresponding force becomes

$$F=F_U+F_S=-\left(M_w\ddot{X}+\frac{M_w}{2}\ddot{q}_1\right)=-M_w\,\ddot{X}-M_s\,\ddot{q}_1 \tag{19}$$

where , M_w is the liquid total and M_S is half the liquid mass and expresses the part of liquid mass associated with sloshing motion.

3. SOLUTION FOR HARMONIC EXCITATION

The solution is significantly simplified and semi-analytical results may be obtained when the rigid container undergoes a harmonic motion

$$\dot{X}(t)=U\,e^{-i\omega t}, \tag{20}$$

where U is the velocity amplitude, and ω is the angular frequency of the external excitation source. Assuming steady-state conditions, functions for $\dot{q}_n(t)$ are also assumed harmonic

$$\dot{q}_n(t)=a_n\,e^{-i\omega t} \tag{21}$$

and the following infinite system of linear algebraic equations is obtained

$$\left(-\omega^2[\mathbf{M}]-i\nu\omega[\mathbf{M}]+[\mathbf{K}]\right)\ \{a\}=\omega^2\ U\ \{\gamma\} \tag{22}$$

In the above system, the square matrix [**M**], the diagonal matrix [**K**] and the vector $\{\gamma\}$ are given by Equations (16a), (16b) and (16c), whereas $\{a\}$ is the unknown vector with components a_{2n-1}, n=1,2,3... Note that when $U=0$ and $\nu=0$, the system of algebraic equations (22) is reduced to a homogeneous system (i.e. an eigenvalue problem), which is identical to the one obtained by Evans & Linton[3]. An estimate of the externally induced sloshing effects on the overall response can be obtained from the computation of the added mass coefficient C_a and a measure of the dissipation mechanism is offered by the dimensionless damping coefficient C_v:

$$C_a=\mathrm{Re}\left[\frac{F_S}{F_U}\right] \qquad \text{and} \qquad C_v=\mathrm{Im}\left[\frac{F_S}{F_U}\right] \tag{23}$$

In the above expressions, Re[ ] and Im[ ] denote the real and the imaginary part of the F_S/F_U ratio respectively. Considering only the first two terms of the expansion (N=1) closed form solutions can be derived for the mass and damping coefficients.

4. NUMERICAL RESULTS AND DISCUSSION

In Figures 2 and 3 the corresponding results are shown in terms of the added mass coefficient C_a and the dimensionless damping coefficient C_v for a unit-radius half-full horizontal cylinder (R=1) for different values of external excitation frequencies. Three different values of the damping parameter ν are considered, namely 0, 0.34 and 0.68 (g=9.81). Figure 2 shows that for the case of zero damping, the response is characterized by large increases in the added mass coefficient C_a in the vicinity of resonant frequencies. There is a sign reversal in C_a at each resonant frequency. When $C_a<0$ the sloshing force F_S opposes the "uniform motion" force F_U resulting in a reduction of the total force amplitude. The extreme values of the added mass coefficient close to the resonant frequencies are significantly reduced when damping is present, and the resonant effect of the higher natural frequencies almost disappears. The large values of C_a for a wide range of excitation frequencies indicate the significant effects of hydrodynamic sloshing on the overall response. Figure 3 presents the corresponding results for the dimensionless damping coefficient C_v, which exhibits a peak near the first resonant frequency, and much smaller peaks for the higher resonant frequencies. When the damping parameter value is increased, the peaks become smaller and smoother.

The response of a horizontal-cylindrical container, with radius R=1m and length L=6m containing a liquid of density ρ=1000kgr/m^3, under the seismic ground motion of Kozani, Greece, in 1995, is considered (Figure 4). The linear system of ordinary differential equations is integrated in time by implementing a fourth-order Runge-Kutta scheme in Matlab programming, and the time step Δt is chosen equal to 0.005 sec. A truncation size N equal to 2 is considered in this analysis. Damping is expressed through the damping ratio $\xi_S=(\nu/2)\sqrt{8R/3\pi g}$. The damping effects are demonstrated clearly in Figure 5, where the time history of the generalized coordinates $q_1(t)$ for zero damping (ν=0) and for 10% damping are depicted. The presence of damping results in a significant attenuation of the $q_1(t)$ value. Figures 6a and 6b show the forces associated with sloshing (F_S) for a half-full cylinder subjected to the Kozani earthquake and for damping ξ_S equal to 0% and 10%, respectively. Finally Figures 7a and 7b show the total forces F for the container including sloshing and uniform motion.

5. CONCLUSIONS

A mathematical model is developed for externally induced liquid sloshing in half-full horizontal cylindrical containers. The velocity potential is split in two parts, a "uniform motion" potential (trivially obtained) and a potential associate with sloshing. In this configuration, the problem formulation is not separable and the general solution of the sloshing potential is written as a series expansion of arbitrary time functions and its associated non-orthogonal spatial functions. Furthermore, viscous damping effects can be taken into consideration through an appropriate modification of the dynamic free surface boundary condition. The formulation results in a system of linear differential equations, which is solved numerically.

The present formulation enables the prediction of sloshing effects under any form of external excitation, in a simple and efficient manner. In the case of harmonic excitation, the results are expressed in terms of the added force coefficient and the dimensionless damping coefficient. It was found that convergence in terms of truncation size is less rapid in the vicinity of resonant frequencies, and that the presence of damping diminishes resonant effects. Finally, the response of a spherical vessel subjected to a real seismic event is examined, and the total horizontal force acting on the container is calculated.

ACKNOWLEDGMENTS

This work has been partially supported by the Earthquake Planning & Protection Organization (EPPO), Athens, Greece. The authors would also like to thank Dr. V. Lekidis and Dr. Ch. Karakostas, Institute of Engineering Seismology & Earthquake Engineering (ITSAK), Thessaloniki, Greece, for providing the seismic input data of the Kozani earthquake.

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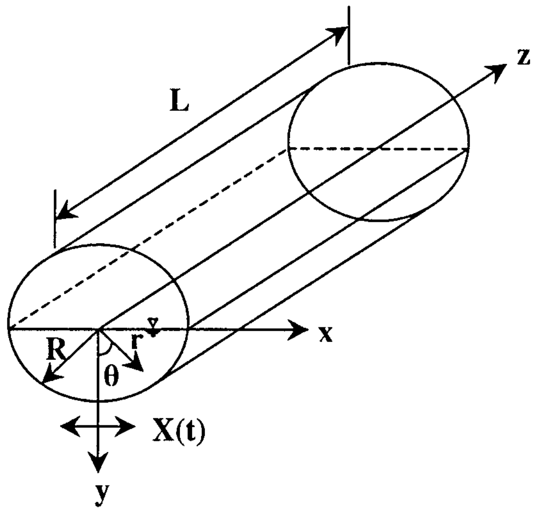


Figure 1: Configuration of half-full horizontal cylinder container under transverse excitation X(t).

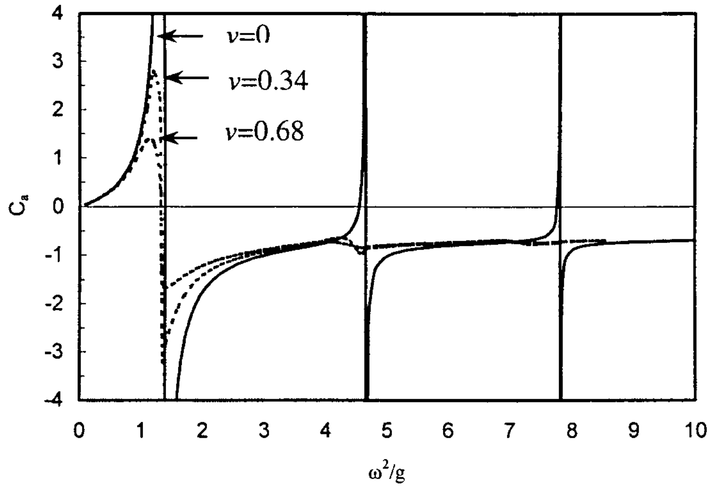


Figure 2: Converged value of C_a in terms of external excitation frequency (ω^2/g) for N=50, (R=1).

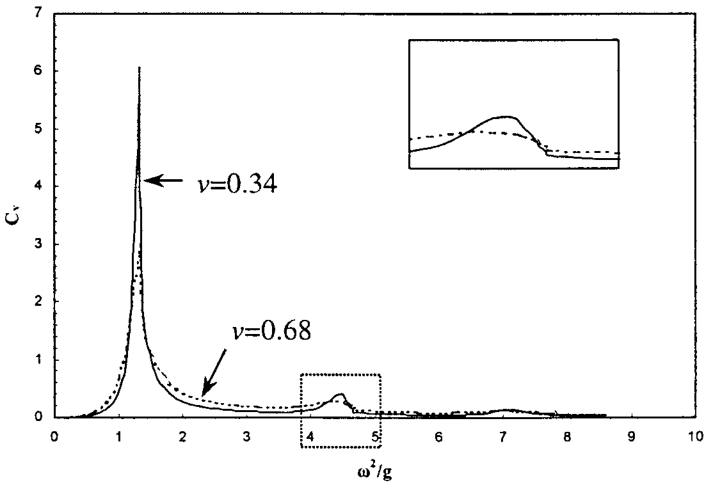
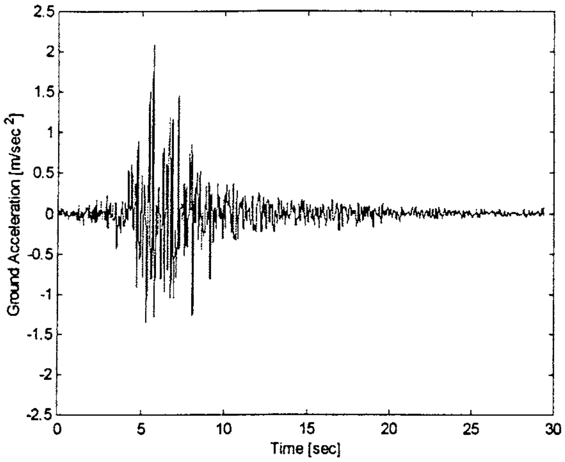


Figure 3: Converged value of C_v in terms of external excitation frequency (ω^2/g) for N=50, (R=1).



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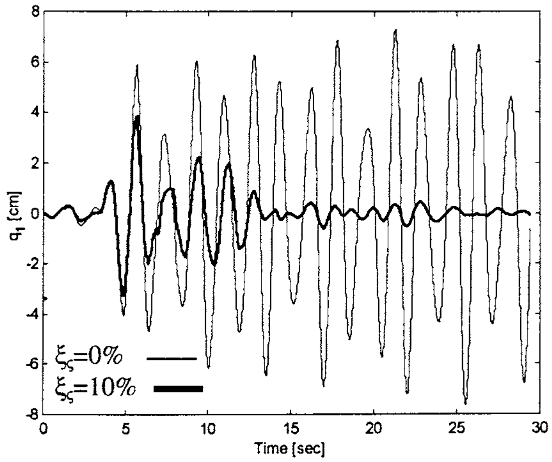

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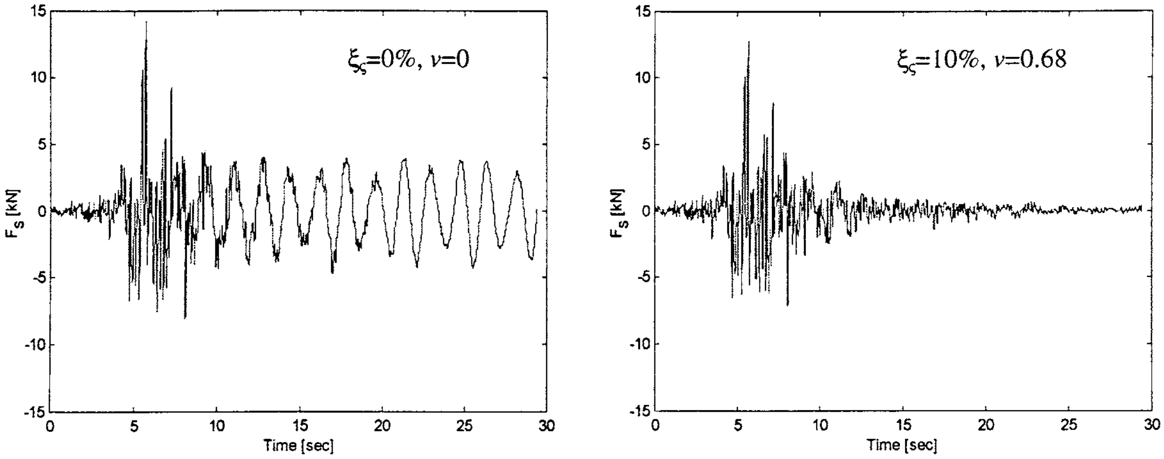

Figure 6: Forces associated with sloshing for an horizontal cylinder of radius R=1m and length L=6m with and without damping effects (g=9.81m/sec^2, ρ=1000kgr/m^3, N=2).

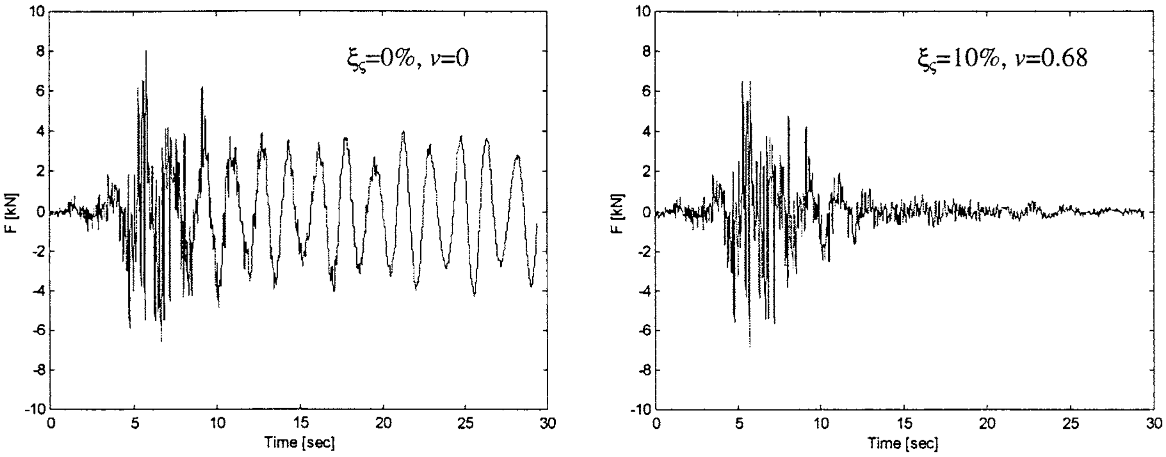

Figure 7: Total forces for a horizontal cylinder of radius R=1m and length L=6m with and without damping effects (g=9.81m/sec^2, ρ=1000kgr/m^3, N=2).

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12th European Conference on Earthquake Engineering
Paper Reference 474 (quote when citing this paper)

SLOSHING EFFECTS IN SPHERICAL VESSELS AND THEIR SUPPORTS

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ABSTRACT

The present work investigates the response of a half-full spherical pressure vessel (tank) under earthquake excitation. The fluid motion due to the free surface usually referred to as "sloshing" is examined in detail with respect to its influence on the response of the tank/support system, in terms of the base shear force and overturning moment. The present study adopts a semi-analytical solution of sloshing in spherical vessels, and proposes a simplified mechanical model to describe the vessel behavior, including the flexibility of its supports. Two case studies from actual industrial applications are considered. In those vessels, the importance of including sloshing effects is examined. Furthermore, the maximum base shear and overturning moment obtained through the direct analysis of a real seismic event are compared with the corresponding values from a spectral analysis procedure.

Keywords: Sloshing, seismic design, pressure vessel, spherical tank, industrial facilities design.

INTRODUCTION

The presence of a free surface in partially filled liquid containers allows for fluid motions, which in turn may influence container's response and structural integrity in the case of a strong earthquake motion. This phenomenon, referred to as "liquid sloshing", is generally caused by external tank excitation, and due to its importance it has been investigated extensively in previous research works. In general, past works contributed on the computation of the sloshing frequencies and the corresponding modes for tanks of various geometries, as well as on the evaluation of the forces and moments exerted on the tank by the sloshing hydrodynamic pressure. Among other significant contributions on the seismic response of liquid storage tanks, it is worth mentioning the works of Housner [1], Haroun and Housner [2], and Veletsos et al. [3], [4], which form the basis for the API provisions (API 650 - Appendix E) for vertical cylindrical tanks [5]. Moreover, the works of Fisher, Rammerstorfer and Scharf [6], [7] have contributed towards the provisions regarding sloshing in vertical cylindrical tanks of Eurocode 8 (EC8 – part 4.3 – Appendix A) [8]. The recent papers by Malhotra et al. [9] and Pandohi-Mishre et al. [10] describe the application of Eurocode 8 on vertical cylindrical tanks, and include useful design examples. The reader is also referred to the review paper of Rammerstorfer et al. [11] for a complete presentation of the earthquake response of vertical cylindrical tanks. In addition, to the above analytical/numerical works, notable experimental contributions on this subject have been reported by [12] and [13].

The above studies have focused on orthogonal and cylindrical (vertical) tanks. On the other hand, spherical tanks are very common in chemical plants and refineries, and they are employed as storage vessels for liquefied petroleum gas (LPG), liquid propane, propylene and LNG. Those tanks are not directly connected to the ground, but there exists a support system with vertical legs and X-braces (Figure 1).

It is important to note that little information exists regarding the seismic response of spherical tanks. In an early work, Budiansky [14] has computed the sloshing frequencies of a spherical tank for arbitrary liquid depth, as well as the tank response under an external excitation. In that work, Budiansky applied conformal mapping on the spherical configuration, resulting in an integral equation for calculating the sloshing eigenfrequencies, eigenmodes, as well as forces and overturning moments. The sloshing frequencies of half-full spherical tanks and for horizontal cylinders have been calculated by Evans & Linton [15] using an analytical methodology. The convergence of solution was discussed, and the computed eigenfrequencies were compared successfully with those reported by previous researchers.

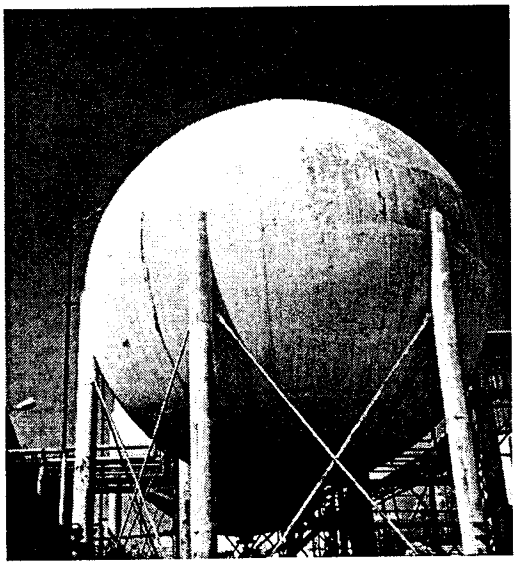

Figure 1: Spherical tank with vertical legs and X-braces.

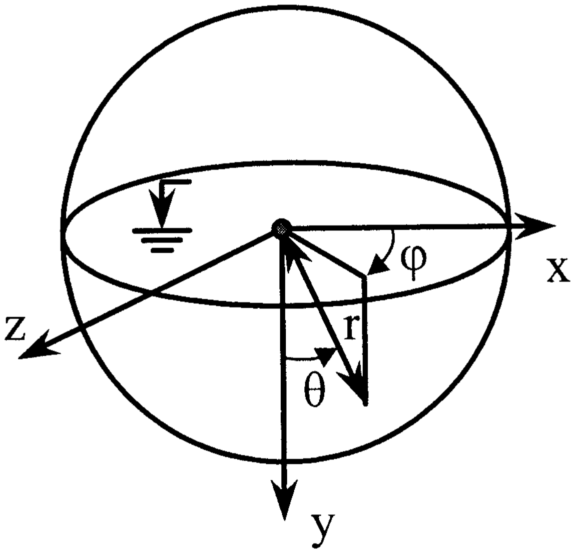

Figure 2: Configuration of half-full spherical container-spherical coordinates.

For design applications, the corresponding American Petroleum Institute (API) standard (API 650) specifies that the tank support design should include sloshing effects, but its guidelines refer exclusively to vertical cylindrical tanks [5]. The New Zealand Earthquake Engineering Society has published design recommendations for the seismic design of liquid storage tanks, including the effects of sloshing [16]. Nevertheless, most of this work was directed towards the design of cylindrical vertical tanks, and very limited information was provided for elevated spherical tanks. The recommendations provide the first sloshing frequency for a spherical vessel and suggest that an equivalent cylindrical tank may be considered to calculate sloshing forces [16]. Similarly, the corresponding provisions of Eurocode 8 (part 4) are quite extensive for cylindrical (vertical) tanks, but very limited for spherical tanks [8], adopting the New Zealand recommendations [16].

In a recent paper [17], the authors have extended the methodology of Evans and Linton [15] to include half-full-tank external excitation, and computed sloshing forces. The present paper investigates the effects of sloshing on the earthquake response in half-full spherical tanks, based on a simplified version of the analytical sloshing solution presented in [17]. The tank response is examined in terms of the resultant base shear force and the corresponding overturning moment.

GENERAL SOLUTION OF SLOSHING IN HALF-FULL SPHERES

Irrotational flow and incompressible fluid are considered. It has been further assumed that the tank is non-deformable, and the amplitude of sloshing waves is small. Spherical pressure vessels are quite thick (to resist to high internal pressure) and, therefore, neglecting shell deformation is a reasonable simplification. Considering tank excitation in one direction (say x), these assumptions result to the following governing equation for the total potential φ_T:

$$\nabla^2 \varphi_T = 0 \tag{1}$$

with boundary conditions

$$\frac{\partial \varphi_T}{\partial n} = \dot{X} \text{ at } r=R \text{ (tank wall) and } \quad \frac{\partial^2 \varphi_T}{\partial t^2} - g\frac{\partial \varphi_T}{\partial y} = 0 \quad \text{at } y=0 \text{ (free surface)} \tag{2}$$

where X is the displacement of the rigid tank, R its radius and g is the gravity acceleration (see Figure 2). The above boundary conditions imply that the velocity at the tank wall equals the motion of the tank, and that a linearized dynamic boundary condition governs the free surface behavior. Following analytical solutions for other configurations (the reader is referred to paper [18] for rectangular tanks and to paper [19] for vertical cylindrical tanks), it is assumed that the solution consists of two parts: (a) the impulsive part φ_w, i.e. the liquid follows the tank motion X(t) as a rigid body

$$\varphi_w = \dot{X}\, r \cos\psi \sin\theta \tag{3}$$

and (b) the sloshing part φ, which represents the fluid motion relative to the tank due to sloshing, so that

$$\varphi_T = \varphi_w + \varphi \tag{4}$$

The following series solution is assumed for the sloshing part [16], [17]

$$\varphi = -\sum_{n=1}^{\infty}[\, \dot{q}_{2n-1}\, r^{2n-1}\, P^{1}_{2n-1}(\mu) + \dot{q}_{2n}\, r^{2n}\, P^{1}_{2n}(\mu)\,] \cos\psi \tag{5}$$

where $P^n_m(\mu)$ is the associated Legendre function, $\mu=\cos\theta$, and the q_i's are unknown functions of time. Applying the boundary conditions, the following form of equations is obtained:

$$\bar{\mathbf{M}}\,\{\ddot{\bar{q}}_n\} + \bar{\mathbf{K}}\,\{\bar{q}_n\} = -\ddot{X}\,\frac{2R}{3g}\,\{\gamma_m\} \tag{6}$$

where

$$\bar{q}_n = q_{2n-1}$$

$$\bar{M}_{mn} = M_{mn} \frac{2n}{2n+1} \frac{R^{2n-1}}{g} \tag{7}$$

$$\bar{K}_{mm} = K_{mm}\,(2m-1)\,R^{2m-2} \quad \text{and } K_{mn} = 0 \quad \text{if } m \neq n$$

$$m = 1,2,3,\ldots\ldots \; n = 1,2,3,\ldots\ldots$$

and

$$K_{mm} = \int_0^1 P^1_{2m-1}(\mu)\, P^1_{2m-1}(\mu)\, d\mu$$

$$M_{mn} = \int_0^1 P^1_{2n}(\mu)\, P^1_{2m-1}(\mu)\, d\mu \tag{8}$$

$$\gamma_m = \int_0^1 P^1_2(\mu)\, P^1_{2m-1}(\mu)\, d\mu$$

If the tank motion X is zero and considering a harmonic solution for the q_i's with respect to time

$$q_i(t) = \alpha_i\, e^{-i\omega t} \tag{9}$$

the sloshing eigenfrequencies of a spherical vessel are obtained, if non-trivial solutions for the α_i's are sought. The eigenfrequencies are shown in Table 1.

Modes	1	2	3	4
Eigenfrequencies (rad/sec)	3.9121	7.1939	9.1339	10.705

Table 1: The first four sloshing eigenfrequencies of spherical container (R=1).

It is quite important to note that the eigenvalues depend on the truncation of equation (5) and this is because the functions employed in equation (5) are not mutually orthogonal. In Figure 3 the variation of the first (fundamental) eigenfrequency ω_1 is shown in terms of the truncation size N ($n \leq N$). Note that three terms (N=3) are necessary so that a very good accuracy is achieved. The total force F_T applied by the liquid to the tank is obtained by the integration of pressure on the tank wall in the direction of the excitation (say direction x). This force has two components, one "impulsive" F_w and one "sloshing" F_s:

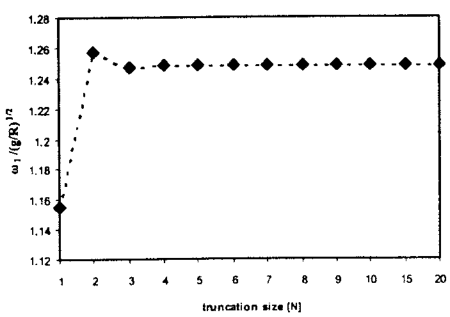

Figure 3: Variation of $\omega_1/\sqrt{g/R}$ with respect to truncation size N for the spherical container.

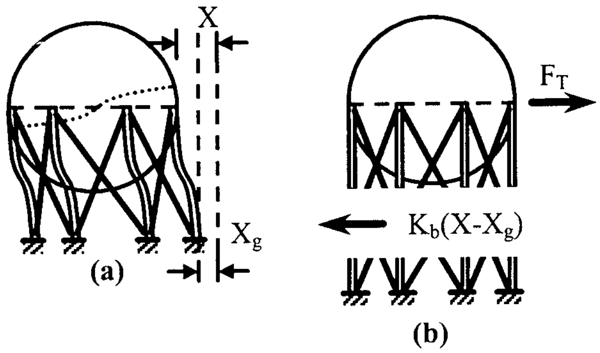

Figure 4: (a) tank motion and ground motion, (b) equilibrium of forces.

$$F_T = F_w + F_{s\ell}$$
$$F_T = \int_{B_l} p_w\,(\mathbf{n}\cdot\mathbf{e}_x)\,dB + \int_{B_l} p_{s\ell}(\mathbf{n}\cdot\mathbf{e}_x)\,dB \tag{10}$$

where

$$p_w = -\rho\,\frac{\partial \varphi_w}{\partial t} \quad \text{and} \quad p_{s\ell} = -\rho\,\frac{\partial \varphi}{\partial t} \tag{11}$$

It is easy to show that the impulsive component is

$$F_w = -M_w\,\ddot{X} \tag{12}$$

where M_w is the total liquid mass ($M_w = \frac{2}{3}\rho\pi R^3$), which was actually expected, because in the impulsive motion every liquid particle follows the motion of the tank. Another important issue is the location of the above forces. Since the pressure is always normal to the tank wall, those forces are applied on the center of the tank. This observation is applicable to spherical tanks filled up to an arbitrary depth (not only half-full) and it is important in order to obtain the corresponding overturning moment

The above formulation and solution assumes that the motion of the tank X(t) is known. In the case of a spherical tank supported by a system of legs and X-braces, only the ground motion $X_g(t)$ is known, whereas X(t) is unknown and should be calculated. The additional equation required to determine X(t) is the equilibrium of forces at the base: the total horizontal inertia force F_T is equilibrated by the support force F_b (Figure 4). This means that

$$F_T - F_b = F_T - K_b\cdot(X\text{-}X_g) = 0 \tag{13}$$

where K_b is the stiffness of the leg/bracing support system, and $X\text{-}X_g$ is the relative displacement of the tank with respect to the ground.

SIMPLIFIED 2DOF MECHANICAL MODEL

The problem formulation is simplified if one term of the series solution is considered only [equation (5) with n=1] and the liquid-tank system is reduced in a 2DOF system. More specifically, it is assumed that the sloshing motion of the liquid with respect to the tank is given by the following approximate expression, instead of equation (5):

$$\varphi(r,\theta,\psi,t) = -[\dot{q}_1\, r\, P_1^1(\cos\theta) + \dot{q}_2\, r^2\, P_2^1(\cos\theta)]\cos\psi \tag{14}$$

Using the above-simplified expression and applying the boundary conditions, the system of ordinary differential equations (6) reduces to only one equation:

$$\ddot{q}_1 + \left(\frac{4g}{3R}\right)q_1 = -\ddot{X} \tag{15}$$

which a linear oscillator equation. The frequency of the oscillator $\omega_s = \sqrt{4g/3R}$ is an approximation of the first sloshing frequency ω_1 (Figure 3). Furthermore, a simple closed-form expression is obtained for the sloshing force:

$$F_{s\ell} = -\left(\frac{M_w}{2}\right)\ddot{q}_1 = -M_s\,\ddot{q}_1 \tag{16}$$

where M_s is half the liquid mass and it is referred to as sloshing mass. It can be shown that the total force based on one sloshing mode is a good approximation of the corresponding force assuming the complete solution (see eq.(5)). The equilibrium of the forces implied by equation (13) needs also be considered. In this equation, the total horizontal force is to be equilibrated by the elastic forces of the supports and, therefore, the impulsive force should include the mass of the steel tank. Conducting the appropriate integration for the sloshing part as indicated by equations (10), (11), the total force is readily obtained:

$$F'_T = F_{s\ell} + F_w + F_{shell} = -(M_w + M_{shell})\ddot{X} - \frac{M_w}{2}\ddot{q}_1 \tag{17}$$

Inserting the above result in equation (13), the following equation is obtained

$$-M_s\,\ddot{q}_1 - (M_w + M_{shell})\ddot{X} = K_b\,(X - X_g) \tag{18}$$

Equations (15) and (18) constitute a system of linear ordinary differential equations in terms of q_1 and X:

$$\begin{aligned} &M_s\,\ddot{q}_1 + K_s\,q_1 + M_s\,\ddot{X} = 0 \\ &M_s\,\ddot{q}_1 + (M_w + M_{shell})\ddot{X} + K_b\,(X - X_g) = 0 \end{aligned} \tag{19}$$

where $K_s = \omega_s^2 M_s$. Those equations are written in an alternative form, considering the following change of variables:

$$\begin{aligned} &u_1 + X_g = q_1 + X \\ &u_2 + X_g = X \end{aligned} \tag{20}$$

and the final form of the equations becomes

$$\begin{aligned} &\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} K_s & -K_s \\ -K_s & K_s + K_b \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = -\begin{bmatrix} m_1 \\ m_2 \end{bmatrix} \ddot{X}_g \\ &\mathbf{M}\{\ddot{u}\} + \mathbf{K}\{u\} = -\mathbf{M}\,\{1\}\,\ddot{X}_g \end{aligned} \tag{21}$$

where

$$m_1 = M_s \quad \text{and} \quad m_2 = M_w + M_{shell} - M_s \tag{22}$$

The above system of ordinary differential equations (21) has a standard form for a 2DOF structural system with lumped masses.

The model is shown in Figure 5 and it is similar to mechanical models proposed elsewhere for rectangular and vertical-cylindrical tanks [2], [8],[16]. The second DOF (u_2) corresponds to the so-called "convective" motion and the corresponding mass m_2 is the difference between the total moving mass and the sloshing mass. Having computed u_1 and u_2 with respect to time, the forces and the corresponding moments can also be computed.

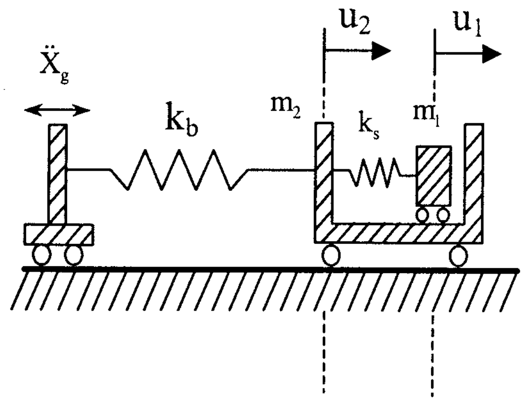

Figure 5: Simple mechanical model for vessel seismic analysis.

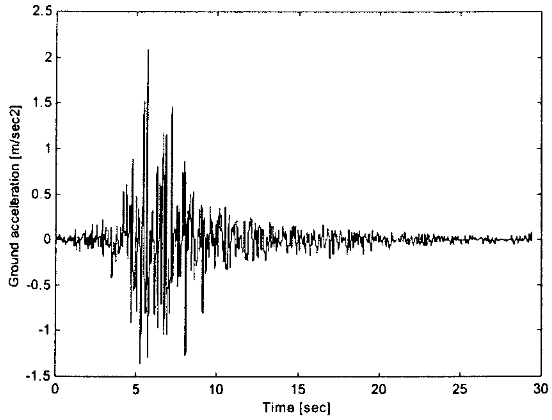

Figure 6: Accelerograph of Kozani earthquake.

NUMERICAL RESULTS

The response of elevated spherical tanks under seismic loading is of particular importance, because of practical applications in the petrochemical industry. Herein, two case studies are examined; a LNG terminal and a propylene terminal, both located in Greece. The stress resultants (i.e. base shear and overturning moment) on the foundation are computed using the simplified 2DOF model formulation and the ground motion record from a recent Greek earthquake, the 1995 Kozani earthquake (Figure 6). The base resultants are compared with the corresponding values when liquid sloshing is not considered. In addition, a spectral analysis is performed according to the general provisions of the Greek Seismic Code [21], using the eigenfrequencies obtained in the course of the present work, and the corresponding stress resultants are also obtained.

First design example

The spherical pressure vessel of this example contains propylene with density equal to 553kgr/m^3. The vessel has internal diameter and thickness 21216 mm and 43 mm respectively. The mass of the liquid when the vessel is half-full is 1.38×10^6 kgr and the mass of the empty steel tank is 0.48×10^6 kgr. Twelve (12) column legs with height h_L=9.2m support the vessel, and they are connected with X-braces. The legs are ∅1160mm×60mm tubular members, whereas the braces are plate members with rectangular cross-section 250mm×35mm and an effective length L_b=8.2 m. To calculate the stiffness of the leg-bracing system, the legs are considered as fixed-fixed sway columns and each X-brace is considered active only with its tensile member. Under those assumptions, the stiffness of the leg-bracing system is calculated as follows

$$K_b = \sum_{j=1}^{N_{legs}} \frac{12EI_L}{h_L^3} + \sum_{k=1}^{N_{braces}} \frac{EA_b}{L_b} \cos^2 a_k \cos^2\beta \tag{23}$$

where EI_L is the bending stiffness of each leg, EA_b is the axial stiffness of each brace, β is the angle of inclination for the braces (60°, same for all braces) and α_k is the horizontal angle between each X-brace and the earthquake direction. First, the eigenvalues ω_1 and ω_2 and the eigenmodes $\{\psi\}_1$ and $\{\psi\}_2$ of the 2DOF model are calculated from a standard eigenvalue analysis:

$$\left[\mathbf{K} - \omega^2\, \mathbf{M}\right]\, \{\psi\} = 0 \tag{24}$$

which is equation (21) with X_g=0. The two eigen-periods T_1, T_2 are very much apart (5.66sec and 0.24sec), showing that the sloshing motion is much "slower" than the structural motion so that the two masses m_1 and m_2 are rather uncoupled. Subsequently, the 2DOF model is analyzed for the Kozani earthquake ground motion, using a standard Runge-Kutta integration of equations (21). The base shear and the overturning moment are obtained as follows:

$$\begin{aligned} V_{base} &= -\, m_1\, (\ddot{u}_1 + \ddot{X}_g) - m_2\, (\ddot{u}_2 + \ddot{X}_g) \\ M_{over} &= h_c\, V_{base} \end{aligned} \tag{25}$$

where h_c is the elevation of the sphere center with respect to the foundation level. For the present example h_c is equal to 14.32m, and the maximum values of V_{base} and M_{over} are 10.21 MN and 146.2 MNm respectively. The variation of V_{base} with respect to time is shown in Figure 7. These values are compared with the corresponding values obtained ignoring sloshing. In this latter case, the vessel-liquid system reduces to a single-DOF oscillator:

$$(m_1 + m_2)\, \ddot{\bar{u}}_2 + K_b\, \bar{u}_2 = -\, (m_1 + m_2)\, \ddot{X}_g \tag{26}$$

where $\bar{u}_2$ is the DOF of the system. The eigen-period of the system is 0.30sec. Solution of the above equation for the ground motion of Figure 6 provides the total base shear and the corresponding overturning moment:

$$\begin{aligned} V_{base}^{(0)} &= -\, (m_1 + m_2)\, (\ddot{\bar{u}}_2 + \ddot{X}_g) \\ M_{over}^{(0)} &= h_c \cdot V_{base}^{(0)} \end{aligned} \tag{27}$$

and the maximum values of $V^{(0)}_{base}$ and $M^{(0)}_{over}$ are 15.19 MN and 217.5 MNm respectively. The variation of $V^{(0)}_{base}$ is also shown in Figure 7. The comparison with the values calculated from equations (24) shows that sloshing has a beneficial effect on the overall response of the tank under consideration.

Alternatively, it is possible to estimate the above base shear and moment using a standard simplified spectral analysis. Having conducted the eigenvalue and analysis, it is straightforward to compute the maximum force vector for each mode (n=1,2), using the spectrum of the Greek Seismic Code [21] for the Kozani area (where the ground motion of Figure 6 was measured). A 2% damping parameter ξ_n is considered for both modes, so that

$$\{F\}_{n,\max} = \mathbf{M}\, \{\psi\}_n\, \frac{L_n}{M_n}\, S_a(T_n, \xi_n) \tag{28}$$

where

$$\begin{aligned} M_n &= \{\psi\}_n^T\, \mathbf{M}\, \{\psi\}_n \\ L_n &= \{\psi\}_n^T\, \mathbf{M}\, \{1\} \end{aligned} \tag{29}$$

and $S_a(T_n, \xi_n)$ is the spectral acceleration value. Considering a maximum ground acceleration equal to 0.24g and type B soil conditions, and assuming that the foundation factor, the importance factor and the behavior factor are all equal to 1, the corresponding values of S_a

are 0.25g and 1.12g for two modes respectively. The force vectors that correspond to the two eigenmodes are combined using the SRSS method (Square-Root-of-the-Sum-of-Squares) [20], so that the total force vector is computed. Finally, the values of V_{base} and M_{over} are computed, equal to 10.25 MN and 146.79 MNm. These values are compared with the more exact values from equation (25), indicating shows the conservativeness of the spectral analysis, with respect to the direct integration of real accelerograms.

Second design example

The second design example consists of determining the base shear and the overturning moment for a spherical terminal, which contains V.C.M with density equal to 850 kgr/m^3. The vessel has internal diameter and thickness 21200 mm and 36.6 mm respectively. The mass of the liquid when the vessel is half-full is 1.06×10^6 kgr and the mass of the empty steel tank is 0.39×10^6 kgr. The vessel is supported by eleven (11) column legs with height h_L=7.515m, connected with X-braces. The legs are ∅945mm×12.5mm tubular members, whereas the braces are plate members with rectangular cross-section 135mm×28mm and an effective length L_b=7.735m. Equations (21) are the governing equations of the system. An eigenvalue analysis is performed first according to equation (24) and the two eigen-periods are 5.66sec and 0.365sec respectively. Subsequently, a direct integration of the governing equations (21) is performed so that the response of the system subjected to the Kozani earthquake is obtained. The variation of V_{base} is shown in Figure 8. In this tank, h_c=11.3m and the maximum values of V_{base} and M_{over} are 6.02 MN and 68.03 MNm respectively. On the other hand, if sloshing is ignored, and assuming that the entire mass is impulsive, the corresponding maximum values of $V^{(0)}_{base}$ and $M^{(0)}_{base}$ are [see equations (26), (27)] 3.84 MN and 43.39 MNm respectively. The variation of $V^{(0)}_{base}$ is shown in Figure 8. This shows that sloshing has a severe effect on the overall tank response. Finally, the tank is analyzed using the spectral analysis [see equations (28), (29]) with 2% damping, and the maximum values of V_{base} and M_{over} are 13.13 MN and 148.49 MNm respectively.

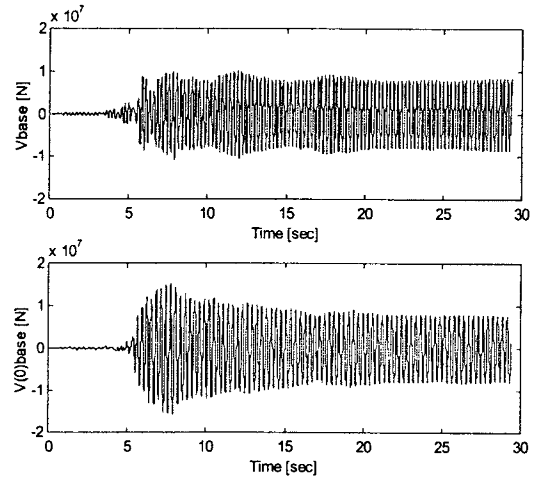

Figure 7: Time histories of V_{base} and $V^{(0)}_{base}$ for the propylene container.

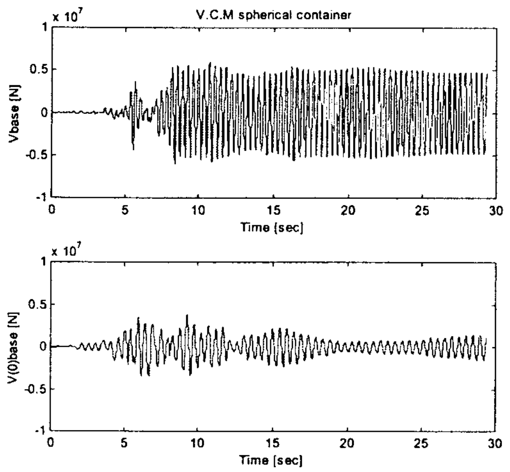

Figure 8: Time histories of V_{base} and $V^{(0)}_{base}$ for the V.C.M. container.

CONCLUSIONS

Based on an analytical series solution of sloshing in half-full spherical tanks, it is possible to develop a simplified model for the analysis of such tanks under earthquake excitation. Using this simple formulation, the seismic analysis of two typical vessels is conducted. The results show that sloshing has a considerable effect on the overall response, especially on the values of base shear and overturning moment. Furthermore, the results from a spectral analysis are compared with the results of the present analysis. The present study is of particular interest to the petrochemical industry, and provides a simple and effective tool for analyzing sloshing effects in half-full spherical liquid-storage vessels. The results are expected to contribute towards better understanding of the sloshing phenomenon and towards safer terminal design.

ACKNOWLEDGEMENTS

The present study was partially supported by the Earthquake Planning & Protection Organization (EPPO), Athens, Greece. The authors would also like to thank Dr. V. Lekidis and Dr. Ch. Carakostas, Institute of Engineering Seismology & Earthquake Engineering (ITSAK), Thessaloniki, Greece, for providing the Kozani 1995 earthquake motion data.

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Longitudinal sloshing effects in half full horizontal cylindrical vessels

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Abstract.

A semi-analytical mathematical model is developed for sloshing effects in half-full horizontal cylindrical viscous liquid vessels, under arbitrary longitudinal external excitation. The velocity potential is expressed in a series form, where each term is the product of a time function and the associated spatial function. In this configuration, the problem is not separable and the associated spatial functions are non-orthogonal. Application of the boundary conditions results in a system of ordinary linear differential equations, which are solved numerically. The motion of a real liquid gives rise to energy dissipation, which is modeled through an appropriate modification of the dynamic free-surface condition. Hydrodynamic forces on the container are calculated for harmonic excitation and for a real seismic event.

Keywords: sloshing, horizontal cylindrical vessel, energy dissipation, seismic response.

1. Introduction

The linear sloshing problem is a typical eigenvalue problem, representing the oscillations of the free surface of an ideal liquid inside a container. The solution provides the natural frequencies of fluid oscillation (sloshing frequencies) and the corresponding sloshing modes, and strongly depends on the shape of the container. For rectangular and vertical-cylindrical containers the sloshing problem can be solved analytically, using separation of variables [1], and the corresponding sloshing modes are mutually orthogonal and uncoupled. For other geometries (e.g. horizontal cylinders or spheres) exact analytical solutions may not be available, and the use of numerical methods becomes necessary.

Sloshing frequencies of horizontal cylinders filled up to an arbitrary height have been determined numerically, through the solution of integral equations [2], [3] or using a variational formulation and Ritz method [4]. Moreover, upper or lower bounds for the values of sloshing frequencies in horizontal cylinders have been obtained [5], [6]. Generally, sloshing analysis in horizontal cylinders filled up to an arbitrary height requires a numerical solution. However, for the particular case of a half-full horizontal cylinder it is possible to develop a semi-analytical solution, as suggested by Evans & Linton [7] for the eigenvalue-sloshing problem.

The present work is aimed at calculating sloshing effects in half-full horizontal cylindrical containers due to external excitation along the cylinder axis, extending the semi-analytical formulation proposed in [7]. In addition, the present formulation considers viscous effects through a simplified approach proposed elsewhere [8]. The velocity potential is expanded in bounded series in terms of arbitrary time functions and their associated non-orthogonal spatial functions, resulting in a system of ordinary linear differential equations, which is solved numerically.

2. Theoretical formulation and solution

The half full horizontal cylindrical vessel of radius R and length L (Figure 1) is non-deformable, and undergoes an arbitrary motion in the direction of the longitudinal axis z with displacement Z(t). The amplitude of free surface elevation is assumed sufficiently small to allow linearization of the problem. It is further assumed that the fluid is inviscid and the flow is described by a velocity potential $\Phi(r,\theta,z,t)$, which satisfies Laplace equation within the fluid volume

$$\nabla^2\Phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}+\frac{\partial^2\Phi}{\partial z^2}=0 \qquad r<R,\ -\pi/2\le\theta\le\pi/2,\ 0\le z\le L \tag{1}$$

subjected to the linearized combined free surface condition

$$\frac{\partial^2\Phi}{\partial t^2}+\nu\frac{\partial\varphi}{\partial t}\pm\frac{g}{r}\frac{\partial\Phi}{\partial\theta}=0, \qquad \text{at } \theta=\pm\pi/2,\ r<R,\ 0\le z\le L \tag{2}$$

where g is the gravitational constant, ν is a viscous parameter, and η=η(r,z,t) is the free surface elevation. Through the second term in Eq (2), a dissipation mechanism is introduced through an elegant and efficient manner [8]. Furthermore, Φ should satisfy the kinematic conditions at the container walls:

$$\frac{\partial\Phi}{\partial r}=0, \qquad \text{at } r=R,\ -\pi/2\le\theta\le\pi/2,\ 0\le z\le L \tag{3}$$

and

$$\frac{\partial\Phi}{\partial z}=\dot{Z}(t) \qquad \text{at } z=0,L,\ -\pi/2\le\theta\le\pi/2,\ 0\le r\le R \tag{4}$$

Subsequently, Φ is decomposed in two parts, as suggested in [9]

$$\Phi(r,\theta,z,t)=f(z,t)+\varphi(r,\theta,z,t), \tag{5}$$

where f(z,t) and φ(r,θ,z,t) are the "uniform motion" velocity potential and the potential related to sloshing respectively. The velocity potential f corresponds to a "rigid body" motion of the fluid, which follows exactly the external excitation source motion:

$$f(z,t)=\dot{Z}(t)\left(z-\frac{L}{2}\right) \tag{6}$$

and satisfies the Laplace equation and the non homogeneous kinematic conditions at z=0 and z=L. Then the velocity potential φ, which represents the relative motion of the fluid particles within the container due to sloshing, should satisfy the Laplace equation within the fluid region and the following boundary conditions:

$$\frac{\partial^2\varphi}{\partial t^2}+\nu\frac{\partial\varphi}{\partial t}\pm\frac{g}{r}\frac{\partial\varphi}{\partial\theta}=-\ddot{Z}(t)\left(z-\frac{L}{2}\right), \qquad \text{at } \theta=\pm\pi/2,\ r<R,\ 0\le z\le L \tag{7}$$

$$\frac{\partial\varphi}{\partial r}=0, \qquad \text{at } r=R,\ 0\le\theta\le\pi/2,\ 0\le z\le L. \tag{8}$$

and

$$\frac{\partial\varphi}{\partial z}=0 \qquad \text{at } z=0,L,\ -\pi/2\le\theta\le\pi/2,\ 0\le r\le R \tag{9}$$

A solution for the unknown function φ is considered in a series form as

$$\varphi(r,\theta,z,t)=\sum_{p=1,3,5}^{\infty}\sum_{n=0}^{\infty}\dot{q}_n^p(t)\, I_n\left(k_p r\right)\cos(n\theta)\cos\left(k_p z\right),\quad r<R,\ -\pi/2\le\theta\le\pi/2,\ 0\le z\le L \tag{10}$$

where $q_n^p(t)$ are unknown arbitrary time functions, $k_p=p\pi/L$, p=1,3,5,...and I_n are the modified Bessel functions of order n. Due to the nature of the external excitation, the above expression is antisymmetric in terms of z (longitudinal direction) and symmetric with respect to the $\theta=0$ plane. The expression of φ is rewritten in the form:

$$\varphi(r,\theta,z,t)=\sum_{p=1,3,5}^{\infty}\sum_{n=0}^{\infty}\left[\dot{q}_{2n}^p(t)\cos(2n\theta)\, I_{2n}\left(k_p r\right)+\dot{q}_{2n+1}^p(t)\cos(2n+1)\theta\, I_{2n+1}\left(k_p r\right)\right]\cos\left(k_p z\right) \tag{11}$$

separating odd and even terms of the series [7]. Substituting Eq (11) into boundary condition Eq (8) and applying the integral operator $\int_0^{\pi/2}\cdots\cos(2m\theta)\,d\theta$ m=0,1,2... the following relations between the even and odd unknown time functions are obtained:

$$\dot{q}_0^p(t)=-\frac{1}{\pi I_0'\left(k_p R\right)}\sum_{m=0}^{\infty}\frac{(-1)^m}{m+1/2}\, I_{2m+1}'\left(k_p R\right)\dot{q}_{2m+1}^p(t), \qquad p=1,3,5,\ldots \tag{12}$$

and

$$\dot{q}_{2n}^p(t)=-\frac{1}{\pi I_{2n}'\left(k_p R\right)}\sum_{m=0}^{\infty}\left(\frac{(-1)^{m-n}}{m-n+1/2}+\frac{(-1)^{m+n}}{m+n+1/2}\right) I_{2m+1}'\left(k_p R\right)\dot{q}_{2m+1}^p(t),\ n>1,\quad p=1,3,5,\ldots \tag{13}$$

Then substituting Eq (11) into boundary condition (7), applying the integral operator $\int_0^L\cdots\cos(k_p z)\,dz$, and using the following identity,

$$1=I_0(x)-2I_2(x)+2I_4(x)-\ldots \tag{14}$$

one obtains

$$\ddot{q}_0^p(t) + \nu\, \dot{q}_0^p(t) - \frac{k_p g}{2} q_1^p(t) = \frac{4}{k_p^2 L} \ddot{Z}(t) \qquad p=1,3,5,\ldots \qquad (15)$$

and

$$\ddot{q}_{2n}^p(t) + \nu\, \dot{q}_{2n}^p(t) - \frac{k_p g}{2}\, q_{2n+1}^p(t) - \frac{k_p g}{2}\, q_{2n-1}^p(t) = \frac{8}{k_p^2 L} \ddot{Z}(t) \qquad n>0,\ p=1,3,5,\ldots \qquad (16)$$

Finally substituting Eqs (12) and (13) into Eqs (15) and (16) respectively and after some mathematical manipulation, the following infinite system of second-order ordinary differential equations is deduced:

$$\left[\mathbf{M}^p\right]\left\{\ddot{q}^p\right\} + \nu\left[\mathbf{M}^p\right]\left\{\dot{q}^p\right\} + \left[\mathbf{K}^p\right]\{q^p\} = -\left\{\gamma^p\right\}\ddot{Z} \qquad p=1,3,5,\ldots \qquad (17)$$

where $[\mathbf{M}^p]$ and $[\mathbf{K}^p]$ are square matrices, $\{\gamma^p\}$ is a vector, with elements

$$M_{nm}^p = \left(\frac{(-1)^{m-n}}{m-n+1/2} + \frac{(-1)^{m+n}}{m+n+1/2}\right) I'_{2m+1}\left(k_p R\right), \qquad n=0,1,2,\ldots,\ m=0,1,2,\ldots,\ p=1,3,5,\ldots \qquad (18)$$

$$\left[\mathbf{K}^p\right] = \frac{k_p \pi}{2} g I'_{2n}(k_p R) \begin{bmatrix} 1 & 0 & \ldots & \ldots & \ldots & 0 \\ 1 & 1 & 0 & \ldots & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & 0 & 1 & 1 & 0 \\ 0 & \ldots & \ldots & 0 & 1 & 1 \end{bmatrix}, \qquad p=1,3,5,\ldots \qquad (19)$$

$$\gamma_0^p = \frac{4\pi}{k_p^2 L} I'_0\left(k_p R\right), \qquad \gamma_n^p = \frac{8\pi}{k_p^2 L} I'_{2n}\left(k_p R\right) \qquad n=0,1,2\ldots,\ p=1,3,5,\ldots \qquad (20)$$

and $\left\{q^p\right\}$ is the unknown vector with components $q_{2m+1}^p(t)$, m=0,1,2,... Solution of (17) is performed through a typical time-marching numerical scheme, and leads to the calculation of functions $q_{2m+1}^p(t)$ and their derivatives.

Once the velocity potential φ is calculated, hydrodynamic pressure is computed from the linearized Bernoulli equation and then the total horizontal force acting on the container is obtained. The total force can be expressed as a summation of the "uniform motion" force and the force associated with sloshing

$$F_U = -\rho\int_A \frac{\partial f}{\partial t}\,(\mathbf{e}_z\cdot\mathbf{n})\,dA \qquad \text{and} \qquad F_S = -\rho\int_A \frac{\partial \varphi}{\partial t}\,(\mathbf{e}_z\cdot\mathbf{n})\,dA \tag{21}$$

respectively, where A corresponds to the "wet" area of the two ends of the container (z=0, z=L) and **n** the outer unit vector normal to A. An estimate of the externally induced sloshing effects on the overall response is offered by the added mass coefficient C_a, whereas the dissipation mechanism effects are represented by the dimensionless damping coefficient C_v:

$$C_a = \mathrm{Re}\left[\frac{F_S}{F_U}\right] \qquad \text{and} \qquad C_v = \mathrm{Im}\left[\frac{F_S}{F_U}\right] \tag{22}$$

where Re[ ] and Im[ ] denote the real and the imaginary parts of the F_S/F_U ratio respectively. The solution is significantly simplified and semi-analytical results may be obtained when the rigid container undergoes a harmonic motion. In this case the system of ordinary differential equations is reduced into a linear algebraic system

3. Numerical results and discussion

Some selected results for a half full cylindrical vessel with R=1 and L=π (k_1=1) subjected to harmonic and arbitrary longitudinal excitation are presented for the first longitudinal mode p=1. Damping is expressed through the damping ratio $\xi_S = \dfrac{\nu}{2\omega_S}$ where $\omega_S = \sqrt{\dfrac{k_1\pi g}{4}\dfrac{I_0'(k_1R)}{I_1'(k_1R)}}$ is an approximation of the first resonant frequency. Three different values of the damping ratio ξ_S=0%, 5% and 10% are considered which correspond to damping parameter ν equal to 0, 0.25 and 0.50 respectively.

In Figures 2a and 2b, the values of C_a and C_v are plotted in terms of the external excitation frequency (ω^2R/g). For ν=0 the areas of resonant frequencies are characterized by large increases in the C_a value. When $C_a \le 0$ F_S is out-of-phase with the cylinder displacement (i.e.

the "uniform motion" force F_U) resulting in a reduction of the total force. The C_a values close to the resonant frequencies are significantly reduced when damping is present, and the resonant effect of the higher natural frequencies almost disappears. The large values of C_a for a wide range of excitation frequencies indicate the significant effects of hydrodynamic sloshing on the overall response. The C_v values exhibit a peak near the first resonant frequency, and much smaller peaks for the higher resonant frequencies. When damping is increased, the peaks become smaller and smoother.

The response of the horizontal-cylindrical container is also calculated under a typical seismic event with maximum ground acceleration 0.21g (Kozani, Greece, 1995). The ODE system (18) is integrated through a fourth-order Runge-Kutta scheme in Matlab programming, with a time step Δt=0.005 sec. Figures 3a and 3b show the forces associated with sloshing F_S for ξ_S 0% and 10%, respectively, corresponding to the first longitudinal mode (p=1).

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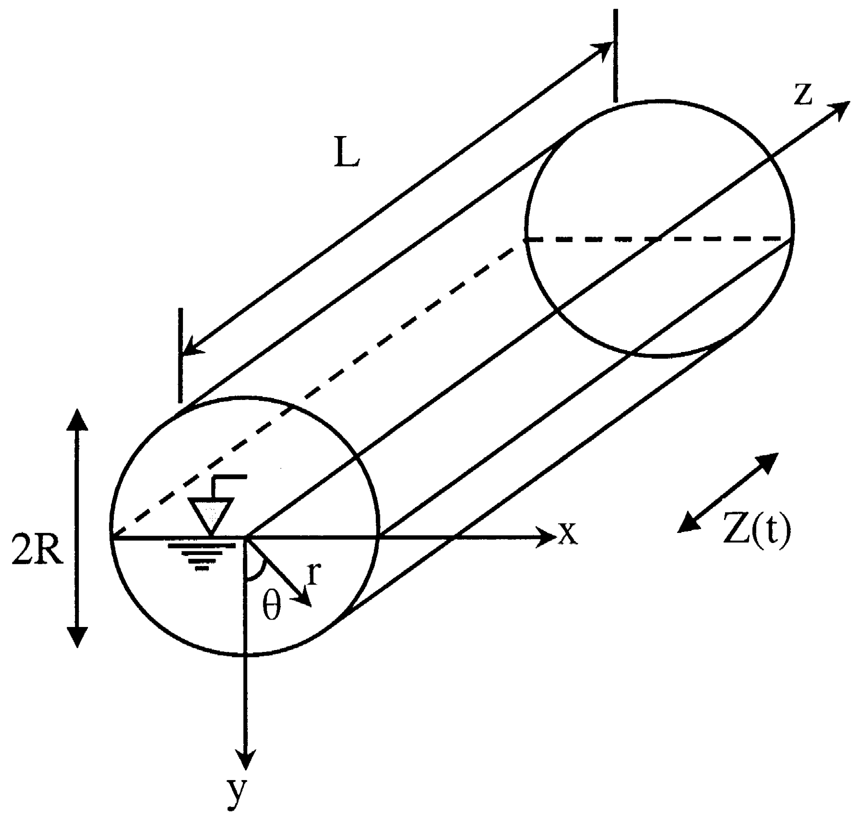


Figure 1: Configuration of half-full horizontal cylindrical container under longitudinal excitation Z(t).

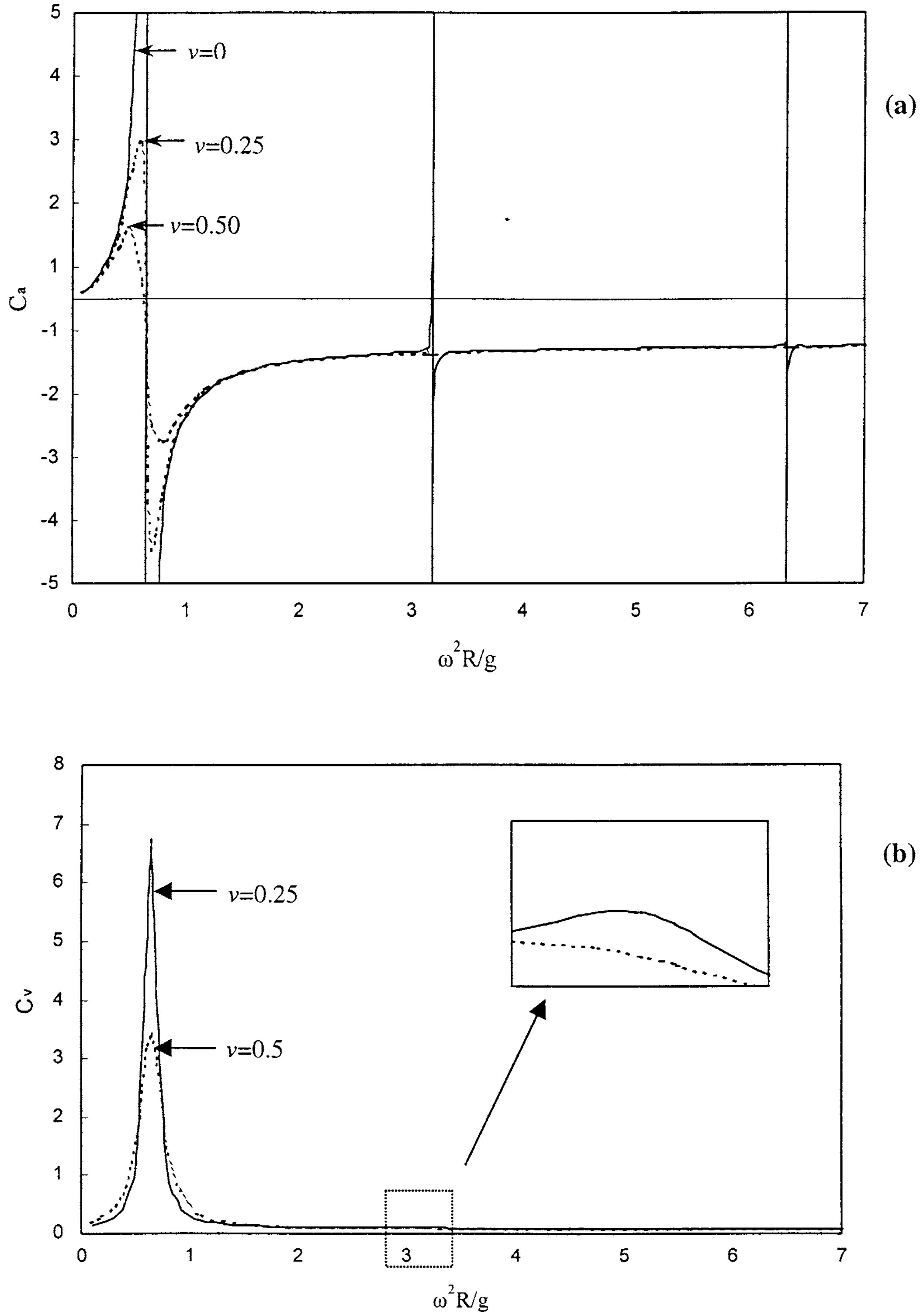
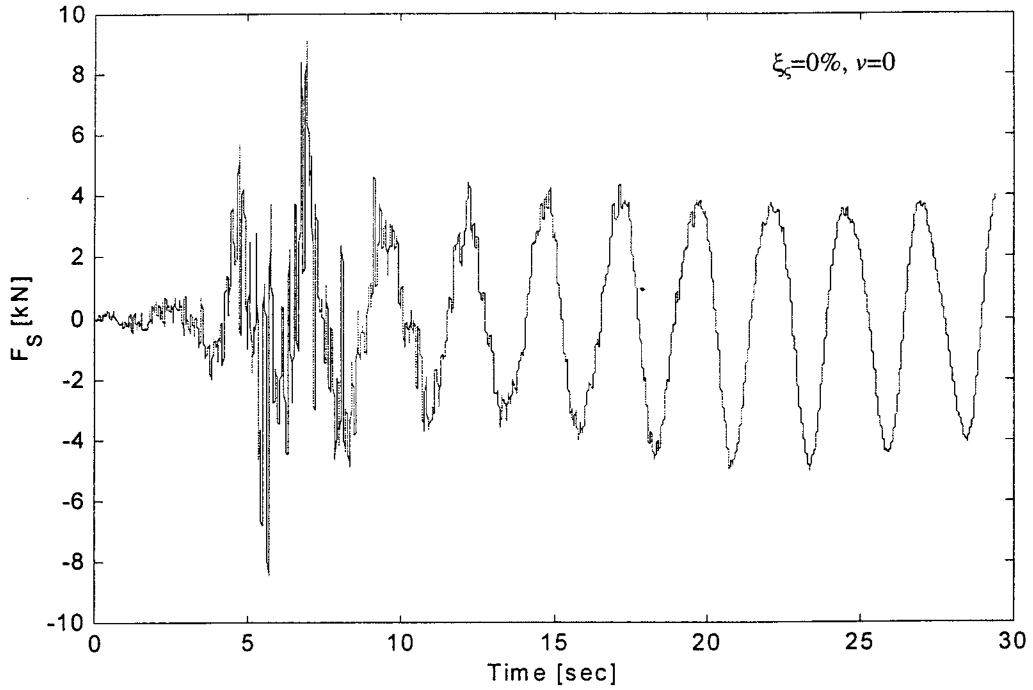
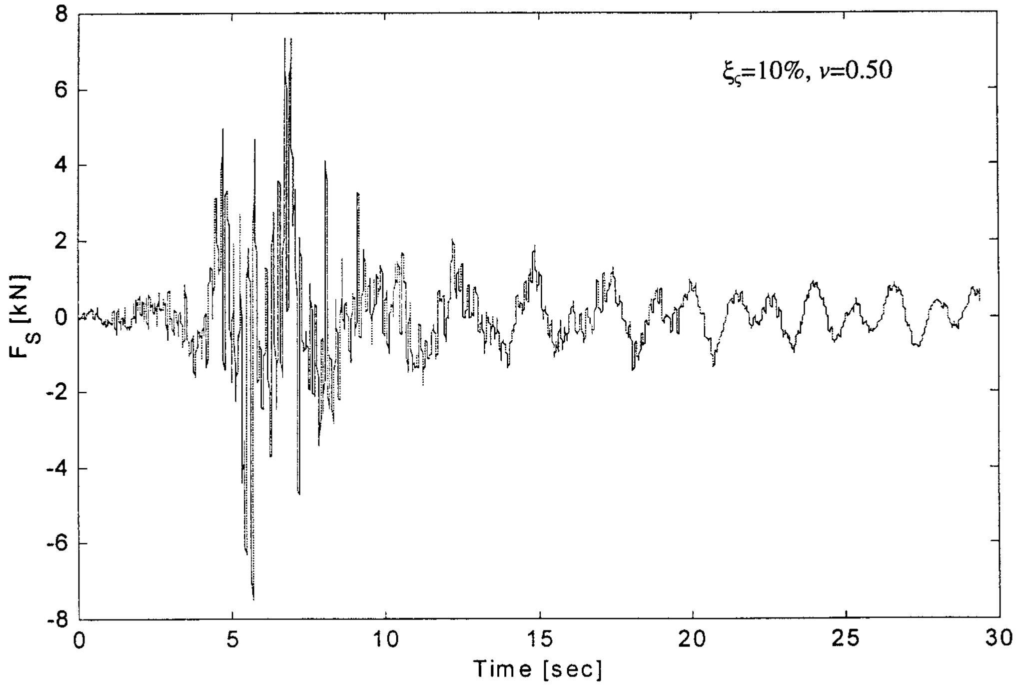


Figure 2: Ca (**a**) and Cv (**b**) values for the first longitudinal mode (p=1) in terms of external excitation frequency (ω^2R/g) (R=1, L=π).



(a)



(b)

Figure 3: Forces associated with the first sloshing mode (p=1) for a half-full horizontal cylinder of radius (R=1, L=π) without **(a)** and with **(b)** damping effects (ξ_S=10%).